Decentralized Income Redistribution and Imperfect mobility

Maria Cubel
Dep. of Hisenda Pública
690, Av. Diagonal
08034 Barcelona. Spain

November 21, 2002

Abstract

We explore the possibility for decentralized redistribution considering a tax competition model where local governments undertake redistribution through the income tax. Two kind of individuals are considered: rich households who are assumed to be immobile and poor households who are imperfectly mobile. We obtain that local redistribution leads to insufficient overall redistribution since there exists a fiscal externality due to migration. This externality increases with household mobility and it can be internalized through a system of matching grants defined by a higher government. The welfare effects of additional immigration are also analysed.

1 Introduction

Most of the contributions in the literature of fiscal federalism and tax competition argue that redistribution should be a function undertaken by the central level of government. For example, consider Musgrave (1971), Oates (1972), Brown and Oates (1987) and Wildasin (1991).

Among those contributions a traditional argument against decentralized redistribution comes from the fiscal externality branch of the literature 1. For example, Wildasin (1991) and also Wellisch (2000) show that decentralized redistribution under perfect mobility produces a fiscal externality since each local government deciding its own redistribution policies affects

---

1See Buchanan and Goetz (1972), Flatters et al. (1974), Stiglitz (1977), Boadway and Flatters (1982) among others.
the policies of the rest. As a result of this external effect redistribution is inefficiently low. Therefore, redistribution should be central or alternatively central government intervention is needed to internalize the fiscal externality.

Pauly (1973) gives an efficiency rationale for decentralized redistribution. He considers two kind of individuals: poor and rich with interdependent utility functions. He considers redistribution as a local public good and he concludes that if preferences for redistribution differ among jurisdictions redistribution should be decentralized. However, Burbidge and Myers (1994a) and Wellisch (1996, 2000) point out the opposite argument. For example, Burbidge and Myers (1994a) argue that when households are mobile it is precisely the difference in preferences for redistribution that causes an inefficient outcome. The reason lies on local governments behaving strategically and offering preferential tax treatment for their favorite type of household which turns out into migration distortions. Only if local governments have equal preferences for redistribution should redistribution be decentralized. Therefore, when preferences for redistribution differ intervention of a central government is Pareto improving with respect to the decentralized solution. However, the reason given by Burbidge and Myers is not the existence of an inefficient population distribution. All in all, the conclusion derived from those studies is that Pauly’s argument can only be supported when households are immobile.

The objective of this paper is to re-examine the consequences of the mobility of population when analysing which level of government should carry out the redistribution function, in doing so we follow the tradition of the studies on fiscal externalities.

We consider a system of jurisdictions (for simplicity two large, strategically-competing regions\(^2\)) where local governments have redistribution responsibilities. Households are imperfectly mobile across regions. In concrete terms we contemplate two sorts of households: completely immobile and imperfectly mobile. Local governments redistribute income between both groups of households using lump sum taxation and lump sum transfers. We use a similar framework as the one developed in Wellisch (2000), however, here we consider that mobile households are not freely mobile but imperfectly mobile since they suffer from home attachment\(^3\). Our results do not contradict

\(^2\)This is a traditional setting in tax competition analysis. See for example, Myers (1990), Wildasin (1991), Wellisch and Wildasin (1996) and Wellisch (2000).

\(^3\)It is quite common in studies of tax competition to consider the existence of two kind of individuals: immobile and perfectly mobile. See for example, Wildasin (1991), Pestieau
but support Wellisch’s analysis offering a more sophisticated scenario.

We obtain that decentralized redistribution produces a fiscal externality if local governments do not take into account the effect of their policies on the other jurisdictions. Hence, the result is that uncoordinated local redistribution produces insufficient redistribution with respect to the social optimum and also results in an inefficient allocation of population between regions due to migration distortions. The fiscal externality increases the marginal cost of regional redistribution and consequently local governments choose a too low level of redistribution with respect to the social optimum. The intervention of a higher level of government can internalize such an externality by defining a system of matching grants such that the social optimum solution is attained.

The distributional and welfare effect of immigration is also explored. We do this analysing the impact of an exogenous increase of population on the equilibrium of the system.

The paper is organized as follows: In section 2 the general setting of the model is developed. In section 3 the conditions for socially optimal redistribution are derived when there is a central planner. In section 4 we explore the uncoordinated Nash equilibrium. In section 5, we derive the conditions for socially optimal redistribution under the intervention of a higher level of government which behaves as Stackelberg leader and implements a system of matching grants in the tradition of a Pigouvian corrective device. In section 6, the effect on welfare from an exogenous change in immigration is analysed. And finally section 7 is dedicated to the conclusions and points out possible extensions for further research. The appendix provides proofs of the results.

2 General Setting

Consider for simplicity only two regions A and B. In every region there exist mobile and immobile households. Each region produces a homogeneous output which is considered as numeraire in the analysis. Production in every jurisdiction $i$ is indicated by a linear homogenous production function $F^i(L_i, N_i)$ which is increasing and concave in all its arguments. Note that production functions may be different in each region, thus allowing for differences in technology. $L_i$ is a fixed factor supplied inelastically in every region...
by a representative immobile resident (which will be indexed by 1)\(^4\). This fixed factor should be understood in a broad sense to include any immobile factor of production such as land, immobile labour\(^5\), natural resources, public infrastructure, etc... \(N_i\) refers to labour from mobile workers (indexed by 2) living in jurisdiction \(i\). Labour is inelastically supplied, every worker produces only one unit of labour and decides in which jurisdiction she prefers to live. Thus workers are mobile between jurisdictions but not perfectly mobile since there is an element of personal attachment to jurisdictions.

Markets are competitive and consequently regional wages are determined by the marginal productivity of labour, \(w_i = F_i\). Local governments redistribute income between mobile and immobile residents implementing a transfer scheme where mobile workers receive a transfer (resp., pay a tax) \(z_i > 0 ( < 0)\)\(^6\) which must be financed by collecting a tax from (giving a subsidy to) immobile households. This transfer could be positive (subsidy) or negative (tax) changing in this way the direction of the local redistribution policy. Hence, the net income of a mobile worker living in region \(i\) is denoted by \(x_{i}^{2} = F_i^{i} + z_i\), and the net income of the immobile worker is equal to \(x_{i}^{1} = F_i - N_i x_{i}^{2}\). The budget constraint of the regional government has already been included in \(x_{i}^{1}\).

Mobile workers are considered imperfectly mobile since they are attached to regions. We use the home-attachment model (in the tradition of DePalma and Papageorgiou, 1988, and Mansoorian and Myers, 1993) which includes complete mobility and immobility as extreme cases\(^7\). Households are heterogeneous with respect to their attachment to a region. We assume one household of each type, denoted by \(n\). \(n\) takes integer values between 0 and \(N\), the total population of mobile workers. The utility function of mobile workers is additively separable with respect to attachment to a region, hence

---

\(^4\)As Wellisch (2000) points out the results of the analysis would not change if the fixed factor were owned by many residents, as long as they are immobile.

\(^5\)We could consider the existence of immobile workers due to age or skill abilities. For example, we could assume that only high-skill workers are mobile and low-skill workers are immobile or the opposite.

\(^6\)When \(z_i > 0\), \(z_i\) can be understood as the net fiscal benefit of mobile households, this includes the totality of tax, transfers and expenditure policies.

\(^7\)This way of modeling imperfect mobility has frequently been used in tax competition models. See for example, Burbige and Myers (1994b), Wellisch (1994, 1995), Mansoorian and Myers (1996, 1997), Caplan et al. (2000) and Sato (2001). Alternatively, imperfect household mobility can also be model by costly migration, where the moving cost may differ among households. See for example, Wildasin (1986), Bucovetsky (2000, 2001),
preferences of type \( n \)-worker are defined as:

\[
V(x, n, r) = \begin{cases} 
U(x) + a(N - n), & \text{if } r = A \\
U(x) + an, & \text{if } r = B
\end{cases}
\]

where \( x_i^2 \) is the net income income of mobile workers, \( i = A \) and \( B \) and \( r \) refers to region. Utility is strictly concave and strictly increasing in \( x_i^2 \). The parameter \( a \geq 0 \) refers to home attachment and measures the degree of heterogeneity in tastes for a region, the degree of household mobility. Hence \( a(N - n) \) (resp. \( an \)) is the attachment (psychological) benefit that type \( n \)-workers obtain when living in region \( A \) (resp. \( B \)). Workers are perfectly mobile if \( a = 0 \). As \( a \) increases workers become less mobile. For some \( a \) sufficiently large to move between regions becomes disadvantageous and workers are immobile. The parameter \( a \) influences only inter-regional migration; it has no effect on individual decision making within each region.

The migration equilibrium can be characterized by the marginal mobile worker who is just indifferent between both regions \( A \) and \( B \) and she does not have incentives to migrate,

\[
U(x_A^2) + a(N - n_A) = U(x_B^2) + an_A
\]

\( n_A \) defined by (1) refers to the marginal worker’s type but it is also the number of mobile workers in region \( A \) after migration, this is \( N_A \). Total employment in the two regions must be equal to total supply of labor therefore, \( N_A + N_B = N \). A type \( n \) worker prefers to live in region \( A \) if \( 0 \leq n < n_A \) and prefers to live in region \( B \) if \( n_A < n \leq N \). (1) implicitly defines \( N_A \) as a function of the transfer levels \( z_i \),

\[
\Psi(z_A, z_B, N_A) = U(x_A^2) - U(x_B^2) + a(N - 2N_A) = 0 \tag{2}
\]

To simplify the analysis we consider quasilinear utility functions so that (2) reduces to,

\[
\Psi(z_A, z_B, N_A) = x_A^2 - x_B^2 + a(N - 2N_A) = 0 \tag{3}
\]

Substituting \( x_i^2 = F_i^A + z_i \) in (3) and implicitly differentiating we derive the migration responses of a change in \( z_i \):
\[ \frac{\partial N_i}{\partial z_i} = -\frac{\partial N_i}{\partial z_j} = -\frac{1}{F_{NN}^i + F_{NN}^j - 2a} = -\frac{1}{D} > 0 \]  \hspace{1cm} (4) \]

where \( D = F_{NN}^i + F_{NN}^j - 2a < 0 \) due to the concavity of the production function. As it would be expected a higher transfer in one region increases the equilibrium labour force in this region and decreases that of the other region. Therefore, there exists an interdependency between the redistribution policies of local governments and the labour market equilibrium in each region.

Given the redistributive policies of each local government \( z_i \) we can define the net incomes of immobile and imperfectly mobile households as,

\[ x_1^i(z_i, z_j) = F_i(L_i, N_i) - N_i F_N^i(L_i, N_i) - N_i z_i \]  \hspace{1cm} (5)

\[ x_2^i(z_i, z_j) = F_N^i(L_i, N_i) + z_i \]

Every local government maximizes a social welfare function (SWF) defined over the net income of its immobile factor owner and the net income of a representative mobile worker, \( W_i(x_1^i, x_2^i) \). This objective function is quite general and it includes the particular case of interdependent utility functions. We assume that regions do not make distinctions between native households and immigrants. Thus immigrants have complete access to redistributive policies and there is a common labour market. We also assume that regions have no means of limiting or controlling migration flows using entrance regulation or similar policies.

\footnote{We have chosen the representative resident approach instead of maximizing total utility (TU hereafter) or average utility (AU hereafter), in order that the SWF does not depend directly on the population size. When this is the case, as in the TU or the AU approach, migration becomes more attractive. For example, if the policy maker maximizes TU he would implement policies which attract immigrants and increase the population size since in that way welfare is also increased. With AU as objective the policy maker would choose policies which attract those households with higher utility. Hence, the kind of objective function might be itself a possible source of inefficiency. See Mansoorian and Myers (1997) for a discussion of the consequences of government objectives when the population is mobile.}

\footnote{It is quite common in the redistribution literature to assume interdependent utility functions between two different households types, such as for example, rich (type 1) and poor (type 2). Rich households derive utility from its own consumption and that of the poor so that its utility is \( U_i^1(x_1^i, x_2^i) \). Maximization of \( U_i^1(x_1^i, x_2^i) \) is equivalent to maximization of \( W_i(x_1^i, x_2^i) \). See for example, Wildasin (1991) and Pauly (1973).}
3 Centralized Redistribution

In this section we derive the necessary condition for efficient labour allocation and for optimal income distribution when there exists cooperation between local governments or equivalently when a central planner is responsible for redistribution. This is our benchmark allocation to which compare the results when redistribution is decentralized. The problem reduces to maximize overall welfare, defined as a linear combination of $W_A$ and $W_B$, or

$$
\delta W_A(x_A^1, x_A^2) + (1 - \delta)W_B(x_B^1, x_B^2) \text{ for } \delta \in [0, 1]\text{ subject to (3) and the feasibility constraint (resource constraint) for the whole federation,}
$$

$$
F^A(L_A, N_A) + F^B(L_B, N - N_A) - x_A^1 - x_A^2 - N_Ax_A^2 - (N - N_A)x_B^2 = 0 \tag{6}
$$

Notice that we do not include the parameter for locational tastes in the objective function, however it is taken into account through the migration equilibrium constraint (3). Any change in location must imply an increase in $W_A$ or $W_B$ such that overall welfare increases. Quoting Wellisch (1994) this is a revealed preference argument: if a change in location did not increase utility, it would not be made. Thus, if the objective function is not maximized, the allocation cannot be Pareto efficient. In other words, the maximization of $\delta W_A(x_A^1, x_A^2) + (1 - \delta)W_B(x_B^1, x_B^2)$ subject to (3) and (6) characterizes a Pareto efficient allocation for a given weight $\delta$.

Formally the social planner’s problem is,

$$
\text{Max. } \delta W_A(x_A^1, x_A^2) + (1 - \delta)W_B(x_B^1, x_B^2) \tag{7}
$$

$$\{x_A^1, x_A^2, x_B^1, x_B^2, N_A\}
$$

st. (3) and (6)

Defining $\lambda_1$ and $\lambda_2$ as the Lagrange multipliers associated with, respectively, the migration equilibrium constraint (3) and the feasibility constraint (6) we obtain the following first-order conditions (with instruments being optimized shown in parentheses):

$$
(x_A^1) : W^1_{x_A} - \lambda_2 = 0 \tag{8}
$$

$$
(x_A^2) : W^2_{x_A} + \lambda_1 - \lambda_2N_A = 0 \tag{9}
$$

$$
(x_B^2) : W^2_{x_B} - \lambda_1 - \lambda_2(N - N_A) = 0 \tag{10}
$$

$$
(N_A) : -2a\lambda_1 + \lambda_2[(F^A_N - x_A^2) - (F^B_N - x_B^2)] = 0 \tag{11}
$$
where $W^k_{x_i} = \frac{\partial W^k}{\partial x_i}$, $k = 1, 2$.

Let us define $MRS_i = \frac{W^2_{x_i}}{W^2_{x_j}}$ as the marginal rate of substitution between the net income (consumption) of a representative mobile worker and that of the immobile factor owner in region $i$, $i = A$ and $B$. Then using (8), (9) and (10) we obtain the condition for optimal redistribution,

$$MRS_A + MRS_B = N$$

Thus an optimal redistribution requires that the marginal social benefit of increasing the net income of mobile workers in both regions is equal to the marginal social cost. Since all mobile workers within the same region receive equal net income, the overall marginal social cost is just the sum of the number of mobile workers in each region, $N$.

Using (9) and (10) to solve for $\lambda_1$ and $\lambda_2$ and integrating them into (11) we find the condition for the efficient population distribution:

$$(F^A_N - x_A) - (F^B_N - x_B) = 2a \left( \frac{(1 - \delta)N_AW^2_{x_B} - \delta(N - N_A)W^2_{x_A}}{\delta W^2_{x_B} + (1 - \delta)W^2_{x_A}} \right)$$

Given $0 \leq \delta \leq 1$ (13) can be reduced to:

$$-2aN_B \leq (F^A_N - x_A) - (F^B_N - x_B) \leq 2aN_A$$

Notice that for $a = 0$ (13) reduces to the standard condition of efficient population distribution with perfectly mobile households, $(F^A_N - x_A) = (F^B_N - x_B)$. In the perfect mobility framework households would migrate until net incomes are equalized, $x_A = x_B$, recall (3). Therefore, the efficient population condition reduces to $F^A_N = F^B_N$ as in Wellisch (2000).

If households are imperfectly mobile ($a > 0$), there exists, however, a range of efficient allocations. This range starts with $-2aN_B = (F^A_N - x_A) - (F^B_N - x_B)$ for $\delta = 1$ and ends with $(F^A_N - x_A) - (F^B_N - x_B) = 2aN_A$ for $\delta = 0$.

When mobile households are perfectly mobile condition $F^A_N = F^B_N$ implies that redistribution policies ($z_i$) in every region must be the same. However, when we introduce attachment to regions (imperfect mobility) this condition does not apply and transfers could be different.

Let us consider the special case of $\delta = \frac{1}{2}$ where each region has equal weight in the maximization problem. Then (13) becomes,
\[(F_N^A - x_A) - (F_N^B - x_B) = 2a \left( \frac{N_A W_x^2 - (N - N_A) W_x^2}{W_{x_A}^2 + W_{x_B}^2} \right) \tag{14}\]

This expression depends on the regional preferences for redistribution. In the case that regions have equal preferences for redistribution \((W_{x_A}^2 = W_{x_B}^2)\) (14) becomes,

\[(F_N^A - x_A) - (F_N^B - x_B) = a(2N_A - N) \tag{15}\]

Then even when local governments have equal preferences for redistribution the level of transfers \((F_i - x_i = z_i)\) is not equalized between regions. This result derives from the imperfect mobility of workers. In fact, transfers will only be uniform across regions when regions are perfectly symmetrical such that \(N_A = N_B = \frac{N}{2}\) and \(W_{x_A}^2 = W_{x_B}^2\).

### 4 Decentralized Redistribution

Every local government chooses the redistributive policy \((z_i)\) which maximizes the welfare of their own residents taking into account migration responses and taking as given the redistributive policy of the other region \((z_j)\). Thus local governments behave as Nash and decide their redistributive policies non-cooperatively. The local government problem is stated as,

\[
\text{Max. } W_i(x_i^1, x_i^2) \tag{16}
\]

\[
\{z_i\}
\]

\[
\text{st. } (3) \text{ and } (5)
\]

Following Wellisch (2000) we express the first-order condition of this problem as the change in social welfare in region \(i\) measured in terms of real income of the fixed factor (the equivalent variation),

\[
\frac{dW_i}{dz_i} = (MRS_i - N_i)(1 + F_{NN}^i \frac{\partial N_i}{\partial z_i}) - z_i \frac{\partial N_i}{\partial z_i} = 0 \tag{17}
\]

where \(dW_i = MRS_i dx_i^2 + dx_i^1\). Incorporating \(\frac{\partial N_i}{\partial z_i}\) defined by (4) and rearranging terms we obtain the Nash equilibrium first-order conditions for both regions.
\[ MRS_i = N_i - \frac{z_i}{F_{NN}^j - 2a}, \quad i, j = A, B, \quad i \neq j \]  

(18)

The left hand side reflects the benefit of increasing the net income of mobile workers living in region \( i \) by one unit. In other words, the marginal willingness to pay an increase in \( z_i \) is expressed in units of lost income for immobile factor owners. The right hand side is the marginal cost of such a redistributive policy. We can distinguish a direct marginal cost equal to \( N_i \), the number of recipients of \( z_i \), and an indirect cost due to migration, \( \frac{z_i}{F_{NN}^j - 2a} \). This extra term is the additional cost of redistribution due to migrants coming to region \( i \) attracted by the increase in the net income of workers and it is therefore an horizontal externality. Thus, total marginal cost in region \( i \) increases as a result of migration when \( z_i > 0 \) and decreases if \( z_i < 0 \). Since labour is mobile, changes in \( z_i \) modify labour allocation. This effect is captured by \( -\frac{1}{F_{NN}^j - 2a} \) which shows the change in net migration coming from region \( j \) to region \( i \). Notice that with perfectly mobile workers \( (a = 0) \) this term is larger.

**Proposition 1** At the Nash equilibrium, under decentralized redistribution, 
\( z_i (MRS_i - N_i) > 0 \).

For \( z_i > 0 \) the indirect marginal cost of redistribution is positive and therefore we have \( MRS_i - N_i > 0 \). This yields insufficient redistribution since there is an incentive to decrease \( z_i \) to reduce the total marginal cost of redistribution. If \( z_i < 0 \), the opposite occurs.

Using the first-order conditions for both regions we obtain the following expression,

\[ z_A - z_B = (N_A - MRS_A)(F_{NN}^B - 2a) - (N_B - MRS_B)(F_{NN}^A - 2a) \]  

(19)

Setting \( \delta = \frac{1}{2} \) in (13) and comparing it with (19) we deduce that the condition for an efficient population distribution does not hold when local governments behave non-cooperatively. Therefore, migration distortions do appear. Furthermore, the level of redistribution is not at the optimum since the marginal cost of redistribution perceived by the local government is too high \( (z_i > 0) \) or too low \( (z_i < 0) \) and redistribution is respectively suboptimally low or suboptimally high. For example, when \( z_i > 0 \), local governments are aware that an increase in the transfer level attracts mobile workers.
workers to the region and increases the marginal cost of redistribution. Consequently, they do not have incentives to choose the socially optimal level of redistribution.

Decentralized redistribution when we introduce locational tastes for mobile workers yields the expression,

$$MRS_A + MRS_B = N - \left( \frac{z_A}{F_{NN}^B - 2a} + \frac{z_B}{F_{NN}^A - 2a} \right)$$

(20)

where the term in brackets points out the inefficiency due to migration. In concrete terms, the inefficiency comes effectively from tax competition, this is the fiscal external effect that one region imposes on the other.

Let us assume for example, that $z_i > 0$, then any increase in the transfer by one region attracts workers from the other region lowering the cost of redistribution in that other region. Therefore, there is a positive externality because the region increasing transfers in the first place was not aware of the effect of its actions in the other region.

Evaluating the change in $W_j$ with respect to $z_i$ we determine the value of this externality as (see appendix section A),

$$\frac{dW_j}{dz_i} = -\frac{z_j}{F_{NN}^i - 2a}$$

(21)

The external effect on $W_j$ due to a change in the redistribution policy of region $i$ ($z_i$) is positive whenever $z_j$ is also positive since $F_{NN}^i - 2a < 0$. This externality is smaller the larger is the parameter of locational taste, meaning that a higher mobility of households (smaller $a$) reinforces the external effect. Notice also that the external effect is purely fiscal in nature since it vanishes when $z_j = 0$. This positive externality on $W_j$ is not considered by region $i$ when it chooses its redistribution policy and this is the reason why the level of redistribution is too low in region $i$.

5 Central Government Intervention

When we have mobile households, decentralized redistribution does not achieve either an efficient population distribution or an optimal level of redistribution. Local governments choose redistributive policies which provoke migration distortions and migration provokes a fiscal externality which affects in return the redistributive decisions of local governments. In this
section we explore how a superior level of government (referred to as central
government) can correct for those externalities and reach the social optimum
solution of section 3.

Our objective is to define a corrective device which would allow the
central government to introduce incentives such that local governments be-
having non-cooperatively can achieve the socially optimum solution. Redis-
tribution policies would still be the responsibility of local governments but
the central government would help in such a redistribution by developing a
system of interregional grants.

As in the previous sections, we follow the work of Wellisch (2000) and
consider that the central government implements a system of matching
grants $s_i$ to share the cost of redistribution in every region. This system
of grants is financed through a central lump sum tax, $T_i$, on the income of
the immobile factor owner of each region. The budget constraint for the
central government is thus,

$$\sum_{i=A,B} s_i z_i N_i = \sum_{i=A,B} T_i \tag{22}$$

And after including the central tax $T_i$ and matching grants $s_i$ the net
income of the immobile factor owner is,

$$x^1_i = F^i - N_i F^j_N - (1 - s_i) N_i z_i - T_i \tag{23}$$

We consider that the central government behaves as a Stackelberg leader.
Thus each local government chooses the transfer level which maximizes its
welfare function subject to (3) and (5) taking as given the policy of the
central government ($T_i$ and $s_i$) and the transfer level of the other local
government. The first-order condition for this problem is,

$$MRS_i = N_i - \frac{(1 - s_i) z_i}{F^j_{NN} - 2a} - \frac{N_i s_i D}{F^j_{NN} - 2a} \quad i, j = A, B \quad i \neq j \tag{24}$$

Where $D$ is as defined in (4). As before, decentralized redistribution
yields an horizontal fiscal externality. However, a vertical externality also
appears since central and local governments share the tax base of the im-
mobile owner factor. Thus when a local government chooses its transfer it
affects the redistribution policy of the other local government and also the
budget constraint of the central government. In order to achieve the social
optimum the central government must take into account both kind of externalities when choosing its corrective device \( s_i \). The horizontal externality is now defined as,

\[
\frac{dW_j}{dz_i} = \frac{(1-s_j)z_j}{F_{NN} - 2a} - \frac{N_js_jF_{NN}^i}{F_{NN} - 2a} \quad i, j = A, B \quad i \neq j
\]  

And the vertical externality is,

\[
\frac{d(\sum_{i=A,B} T_i)}{dz_i} = s_iN_i + s_iz_i \frac{\partial N_i}{\partial z_i} + s_jz_j \frac{\partial N_j}{\partial z_i} \quad i, j = A, B \quad i \neq j
\]  

The central government chooses \( s_i \) to neutralize the total externality in each region which is just the sum of (25) and (26). Therefore, the matching grant in every region \( (s_i) \) is given by the solution of the following two-equation system,

\[
\frac{dW_B}{dz_A} - \frac{d(\sum_{i=A,B} T_i)}{dz_A} = 0
\]

\[
\frac{dW_A}{dz_B} - \frac{d(\sum_{i=A,B} T_i)}{dz_B} = 0
\]

Solving this system of equations we obtain the socially optimal level of transfers \( (s_i) \) in each region for \( a > 0 \),

\[
s_i = \left[ \frac{z_iF_{NN}^j - z_j\theta_{ij}}{z_i - DN_i} \right] \frac{1}{2a}
\]

where \( \theta_{ij} = F_{NN}^i - 2a \) and \( i, j = A, B \ i \neq j \).

Solving the system of equations defined by (27) and (28) yields \( z_i - z_j = \{ s_i [\bar{z_i} - N_i] - s_j [\bar{z_j} - N_j] \} 2a \). Clearly under perfect mobility between regions \( (a = 0) \) \( z_i = z_j = z \). Likewise, when workers are imperfectly mobile \( (a > 0) \) it is always possible to define \( s_i \) and \( s_j \) such that transfers in each region are equalized and consequently they do not affect locational choices. Hence, \( z_i = z_j \) if \( s_i [z_i - DN_i] = s_j [z_j - DN_j] \). Therefore, central government intervention can achieve the socially optimal level of redistribution and an efficient population distribution.

In the special case of symmetric regions matching grants are identical, (set \( F_{NN}^A = F_{NN}^B = F_{NN}, N_i = \frac{N}{2} \) and \( z_i = z_j \)), (see appendix section B),
\[ s = \frac{z}{z - N(F_{NN} - a)} \]  

(30)

Notice that under imperfect mobility \((a > 0)\) and identical regions the optimal matching grant \(s\) is smaller, \textit{ceteris paribus}, the larger is the parameter \(a\). Thus, increasing households’ mobility increases the external effect and consequently it also increases the socially optimal matching grant.

6 Welfare Impact of New Immigrants

In section 4 we determined the Nash equilibrium under non-cooperative tax-transfer competition when the population was held fixed. Now we explore how the equilibrium conditions (18) change when we allow the number of immigrants to vary exogenously. In doing this we can analyse the impact of new immigrants on regional welfare when local governments behave as Nash.

When population size is fixed as in sections 3, 4 and 5 we are considering migration between regions. That setting could exemplify the case of regions belonging to the same country or it could also refer to different countries from an economic or political union with a common labour market, such as for example the EU. When we allow for exogenous changes in population size, we are taking into account both migration between jurisdictions and net migration from outside the system (exogenous change in population). Thus, if jurisdictions were regions of the same country, then an exogenous change in population would mean migration from outside that country. For example, we could think of an EU country member and migration from other EU country members and from outside the EU. Of course, as it was specified in the general setting of the model, immigrants should have the same rights as natives. Hence, a realistic scenario would be, for example, the incorporation of a new country into the union. In this case, we could determine the impact of an expansion of the EU on the welfare of each member country. On the other hand, if jurisdictions were the member countries or states of an economic or political union, say for example the EU, we would be considering net migration from outside the EU.

We follow the work of Wellisch and Wildasin (1996, henceforth referred to as W-W) who deal with this question using a more complex model with
perfectly mobile workers and capital taxation\footnote{For additional references on models traditionally used in studies of migration see Michel et al (1998) and other studies cited in W-W.}. Without capital the general equilibrium effects of a change in immigration are simplified since we do not take into account the interactions between labour and capital. Hence, an increase in the number of immigrants would cause a change in the labour market equilibrium modifying wages and consequently altering the return to immobile factor owners. Because of the existence of a common labour market both regions will be affected. Furthermore, new immigrants will also modify the budget constraint of the local government since they are new contributors or new beneficiaries. Thus, all in all new immigration will alter local government policies.

Since the purpose of the analysis is to explore the redistributive and welfare impact of new immigrants we apply a comparative-statics analysis to know the effect of immigration on the net income of mobile and immobile households.

To undertake the analysis we need to assume local stability of the Nash equilibrium. Hence, we can determine $z_i$ and $z_j$ taking (18) for each region. Let us define the matrix $A = (a_{ij})$ with elements

\[
a_{ii} = \frac{\partial (MRS_i - N_i + z_i (1/F_{NN}^i - 2a))}{\partial z_i} \quad \text{and} \quad a_{ij} = \frac{\partial (MRS_i - N_i + z_i (1/F_{NN}^i - 2a))}{\partial z_j}
\]

for $i = A, B$ and $i \neq j$. Differentiating (18) we derive

\[
A \begin{bmatrix} dz_A \\ dz_B \end{bmatrix} = - \begin{bmatrix} a_{AB}(F_{NN}^B - a) \\ a_{BA}(F_{NN}^A - a) \end{bmatrix} dN \tag{31}
\]

where $|A| > 0$ and $a_{ii} < 0$ because of local stability of the Nash equilibrium. Then from (31) we obtain,

\[
\frac{\partial z_i}{\partial N} = - \frac{a_{ij}}{|A|} \left[ a_{jj}(F_{NN}^j - a) - a_{ji}(F_{NN}^i - a) \right] \tag{32}
\]

The terms $a_{ij}$ may be positive or negative, and it is therefore not possible to determine the sign of $\frac{\partial z_i}{\partial N}$. However, it is reasonable to consider that the term in brackets is positive (see appendix section C).

\[
a_{jj}(F_{NN}^j - a) - a_{ji}(F_{NN}^i - a) > 0 \tag{33}
\]
Hence, using (32) we can derive the effect of a change of \( N \) on the net income of imperfectly mobile workers \( (x^2_i) \):

\[
\frac{dx^2_i}{dN} = F^i_{NN} \frac{dN_i}{dN} + \frac{dz_i}{dN}
\]

(34)

and on the income of immobile factor owners \( (x^1_i) \):

\[
\frac{dx^1_i}{dN} = -N_i \frac{dx^2_i}{dN} - z_i \frac{dN_i}{dN}
\]

(35)

**Proposition 2** An exogenous change in immigration (change in population size, \( N \)) decreases the equilibrium net income of imperfectly mobile workers, \( \frac{dx^2_i}{dN} < 0 \), regardless of the sign of \( z_i \). The effect on the net income of the immobile factor’s owners is positive, \( \frac{dx^1_i}{dN} > 0 \), when \( z_i < 0 \) and indeterminate when \( z_i > 0 \).

As shown in the appendix (section C) it is reasonable to consider that the net income of mobile workers is reduced when new immigrants come into the region. This result holds regardless of whether mobile workers are receiving a positive or a negative transfer \( (z_i) \). However, the net income of immobile factor owners increases with additional immigration whenever \( z_i < 0 \). This includes the particular case of symmetric regions pointed out in W-W (1996).

To derive the effect of a change in migration on welfare it is convenient to use the equivalent variation, \( dW_i = MRS_i dx^2_i + dx^1_i \) hence,

\[
\frac{dW_i}{dN} = MRS_i \frac{dx^2_i}{dN} + \frac{dx^1_i}{dN}
\]

(36)

After some mathematical manipulation (36) becomes (see appendix section C),

\[
\frac{dW_i}{dN} = -z_i a_{ii} \left[ a_{jj} (F^j_{NN} - a) - a_{ji} (F^i_{NN} - a) \right] \frac{1}{|A| \left( F^j_{NN} - 2a \right)}
\]

(37)

Taking into account local stability of the Nash equilibrium and that the expression in square brackets is positive, the sign of (37) depends upon the
kind of redistributive policy of the region. If new immigrants are net contributors \((z_i < 0)\) welfare increases and if new immigrants are net beneficiaries \((z_i > 0)\) it decreases. In other words, if mobile workers pay taxes new immigrants are welcome since regional welfare increases even though the net income of mobile workers is reduced. However, if mobile workers receive a positive transfer such that the owners of the immobile factor are the ones paying taxes, regional welfare decreases when new immigrants come into the region. Thus, we obtain the same result as in W-W (1996) but without including capital taxation and considering that mobile workers suffer from home-attachment (imperfect mobility). As we have shown, the absence of capital does not affect the final results. This is related to, although not completely explained by, W-W’s finding that in equilibrium the tax on capital should be zero. The introduction of home-attachment in the analysis mitigates the impact that new immigrants might cause on overall welfare. It would be interesting to analyse the effect of additional immigrants on regional welfare when local governments offer public services and there is congestion. In that case, new immigrants coming into the region would pay taxes or alternatively receive a transfer, and at the same time they will increase congestion on the use of the public good. The effect on welfare will depend upon whether immigrants are net contributors or net beneficiaries once the impact they produce on congestion has been taken into account\(^\text{11}\).

**Proposition 3** Regional welfare increases when new immigrants are net contributors and it decreases when new immigrants are net beneficiaries in the receiving region. This effect is larger the smaller is the attachment to home of mobile workers.

This result may have important political implications. If immigrants are net beneficiaries additional immigrants reduce regional welfare. Hence, there might be scope for inter-regional transfers. It might be in the interest of the receiving region to implement a transfer to the mobile households in the other region so that fiscally induced migration is reduced. Thus, inter-regional transfers might be welfare improving purely for fiscal reasons. This

\(^{11}\text{Smith and Webb (2001), using a Hotelling framework, analyse the strategic tax setting between local governments. They consider mobile and immobile households with different incomes and who allocate across jurisdictions in response to differing tax structures and congestable public goods. They conclude that the impact of migration on inequality within each jurisdiction depends upon the level of income of immigrants. High income immigrants increase inequality and middle income immigrants decrease inequality. The impact on overall welfare depends on the weights attached to both groups. When immigrants are low income earners they induce a reduction of taxes and public amenities and both mobile and immobile households are worse off.}\)
would have a straightforward applicability when considering immigration from developing countries to more developed countries. It could also be applied in the context of a country where there exist disparities between regions or to an economic or political union with disparities between member states. For example, in many federal countries like Canada or Germany there exists fiscal equalization systems which explicitly transfer resources from some regions to others. See Wildasin (1994) for a complete discussion.

6.1 Welfare Impact of New Immigrants with Central Government Intervention

In section 5 we showed how a higher level of government defining a system of matching grants can internalize the fiscal externality produced when local governments decide their redistribution policies without cooperation. Our interest now is to evaluate the effect of additional immigrants on overall welfare when there is such intervention and compare the results with those obtained in the previous section.

Following W-W (1996) we assume that the matching grants \( s_i \) are held constant when there is additional immigration. According to (22) this implies that any possible changes in \( z_i \) due to migration should be compensated by modifications of \( T_A \), \( T_B \) or both.

The change in overall welfare from additional immigration is expressed as follows,

\[
\frac{dW_A}{dN} + \frac{dW_B}{dN} = \sum_{i=A,B} (MR_i - N_i) \frac{dx_i}{dN} - \sum_{i=A,B} z_i \frac{dN_i}{dN} \tag{38}
\]

Taking into account that the intervention of a higher level of government produces an optimal level of redistribution and recalling expression (12), the first term on the right hand side of (38) cancels out and the overall welfare effect of an exogenous change in immigration is,

\[
\frac{dW_A}{dN} + \frac{dW_B}{dN} = - \sum_{i=A,B} z_i \frac{dN_i}{dN} \tag{39}
\]

In the particular case of symmetric regions (39) becomes,

\[
\frac{dW_A}{dN} + \frac{dW_B}{dN} = -z \tag{40}
\]
since \( z_i = z_j \), \( \frac{dN_i}{dN} = \frac{\partial N_i}{\partial N} \) and \( \sum_{i=A,B} \frac{dN_i}{dN} = 1 \).

Therefore, as in W-W when regions are identical the effect of new immigrants on overall welfare is reduced to the size of the net income contribution of the immigrants. Notice than when immigrants are net beneficiaries the overall effect is negative.

7 Conclusions

In this paper we have explored the implications of the decentralization of income redistribution under assumptions of imperfect mobility. We started by defining the necessary conditions for socially optimal overall redistribution when there is a central planner or local governments behave cooperatively. Then we analysed the case of decentralized redistribution under imperfect mobility when mobile workers suffer from locational attachment. As pointed out in previous studies (see for example, Wellisch 2000) the result is that when local governments behave non-cooperatively there is an horizontal fiscal externality which produces a suboptimally low level of redistribution and an inefficient population distribution. We find out that the fiscal externality depends upon the parameter of locational tastes. Thus, the external effect is larger the smaller is the attachment to regions. This means that the externality increases as the mobility of workers increases.

A higher level of government can correct this externality implementing a system of matching grants to partially finance the redistribution policies of local governments. This is a kind of Pigouvian subsidy which lowers the cost of redistribution in each region so that the socially optimal overall redistribution is achieved. However, it also reduces the total income in each region. The interrelation between the central and local governments produces also a vertical externality which can be internalized when implementing the system of matching grants. We find out that the degree of household mobility does not affect the central government’s ability to implement such corrective device. However, if this kind of intervention is not possible then imperfect mobility lowers the cost of decentralized redistribution.

We analysed as well the effect on regional and overall welfare of an exogenous change in population size under decentralized redistribution and under the intervention of a central government. We obtained nearly the same results as in W-W (1996) where they deal with this question using a model with capital and labour taxation and mobile workers are perfectly mobile. We find out that the arrival of new immigrants lowers the net
income of mobile workers regardless of the sign of the transfer they receive, $z_i$. Likewise, the net income of the immobile factor owners increases when they receive a positive transfer ($z_i < 0$). And the effect is ambiguous when they pay a tax ($z_i > 0$). When $z_i > 0$ W-W(1996) also obtain an ambiguous effect except for the case of symmetric regions.

The impact on regional welfare depends also on the sign of the transfer $z_i$. When immigrants are net contributors ($z_i < 0$) regional welfare increases and when they are net beneficiaries ($z_i > 0$) it decreases. Under the corrective device of a central government similar results are derived. In the particular case of symmetric regions the overall impact is equal to the size of the net income contribution of the immigrants ($z$).

Future research should be addressed to introduce some dynamics into the analysis. As Wildasin (2000) emphasizes tax competition in a world of imperfect factor mobility is best analyzed in an explicitly dynamic framework. The attachment to specific locations and occupations changes over time, getting stronger with age so that migration tend to decrease over the life cycle. To model imperfect labour mobility in a dynamic framework we could use an overlapping-generations model or alternatively, we could use an adjustment-cost model of employment and migration. An overlapping-generations model would catch the variation of mobility costs due to age hence it is particularly useful in analyzing long-term fiscal policies with important intergenerational effects such as public pensions, health care and public debt. On the other hand, an adjustment-cost model would take into account any change in cost over time that affects mobility and is not necessarily related to age, such as transportation cost, education cost (acquisition of skills), policy related costs etc..., therefore, an adjustment-cost model is probably the best option, see Wildasin (2001) for an example.

Other possible extensions for future research could consider the introduction of: (i) explicit heterogeneity between regions, (ii) distortionary taxation, (iii) labour effects, (iv) interregional transfers, and (v) two types of imperfectly mobile workers. We comment on each of these suggestions in turn.

(i) Heterogeneity Between Regions

The model can be easily modified to explicitly consider heterogeneity between regions. Following Hindriks and Myles (2000) we could assume that there are two regions $A$ and $B$ and that one region is more attractive than the other for some exogenous reasons. People would prefer to live in the more attractive region because for example, there are more possibilities
of earning income or the region has nicer characteristics like climate, cultural scene, location etc... Different attractiveness can be included in the analysis just introducing a parameter ($\phi$) in the utility function of mobile households as follows,

$$V(x, n, r) = \begin{cases} U(\phi x) + a(N - n) & \text{if } r = A \\ U(x) + an & \text{if } r = B \end{cases}$$

If $\phi > 1$ region $A$ is more attractive than region $B$ and other things equal this region will be richer and more populated in equilibrium. This asymmetry between regions should be understood as if living in region $A$ would provide a higher income or some non-pecuniary benefit.

Under the assumption that region $A$ is more attractive than region $B$ the horizontal externality is now,

$$\frac{dW_A}{dz_B} = -\frac{z_i\phi}{F_{NN}^i - 2a} \quad \frac{dW_B}{dz_A} = -\frac{z_j\phi}{F_{NN}^j - 2a}$$

From (41) is obvious that when the transfer to mobile workers is positive ($z_i > 0$) the externality increases in region $A$ and it decreases in region $B$. Therefore, the cost of redistribution rises in region $A$ and diminishes in region $B$. Under intervention of the central government implementing a system of matching grants between regions the total externality can be internalized. We find out that when region $A$ is more attractive than region $B$ the value of the socially optimal matching grant is reduced in region $A$ and increased in region $B$.

(ii) Distortionary Taxation

We can consider distortionary taxation just by assuming that for every $z_i$ paid by the immobile factorb4s owners mobile workers only receive $(1 - \varphi_i)z_i$, where $\varphi_i \in (0, 1)$ is a deadweight loss parameter that measures any economic distortions apart from migration. The deadweight loss raises the cost of transfers and it also modifies the horizontal externality which now is defined as,

$$\frac{dW_j}{dz_i} = \frac{N_jF_{NN}^i(1 - \varphi_i)\varphi_j}{(F_{NN}^i - 2a)(1 - \varphi_j)} - \frac{z_j(1 - \varphi_j)}{F_{NN}^i - 2a}$$

(42)
The first term in (42) is positive and the second term is the external effect described in (21) times \((1 - \varphi_i)\). Under distortionary taxation the horizontal externality is larger than with no distortionary taxation whenever 
\[
z_j \varphi_i < \frac{-N_j F_{NN} (1 - \varphi_i) \varphi_j}{(1 - \varphi_j)} \]
and it is smaller otherwise. From (42) it is clear that the external effect increases with \(\varphi_j\) and it diminishes with \(\varphi_i\). When regions are symmetric this condition reduces to 
\[
z < -NF_{NN}.\]

However, if we model distortionary taxation such that for every pound that a mobile worker receives in transfer the immobile factor’s owners pay \((1 + \varphi_i)\) the horizontal externality becomes,
\[
\frac{dW_i}{dz_j} = \frac{N_i F_{iNN} \varphi'_i}{F_{NN} - 2a} - \frac{z_i (1 + \varphi'_i)}{F_{NN} - 2a} \tag{43}
\]
and it always increases with the deadweight loss parameter.

(iii) Labour Effects

The model can be easily modified to take into account labour effects. As before consider two kind of households: imperfectly mobile workers and immobile factor-biased owners. Let us define the net income of immobile and mobile households in region \(i\) as: 
\[
x_1^i = F_i - w_i h_i - N_i z_i \quad \text{and} \quad x_2^i = w_i h_i + z_i
\]
where \(h_i\) refers to labour supply, \(w_i\) is wage, \(z_i\) is the transfer received by mobile workers and \(N_i\) is the number of mobile workers in region \(i\). The local government constraint has already been included in the definition of \(x_1^i\). Households face a two stage decision: firstly, they chose their labour supply in each region and, secondly they decide in which region they prefer to live. They chose their location comparing their utilities in each region and assuming a fixed regional population size. As before, local governments maximize social welfare by choosing their redistribution policy \((z_i)\) subject to both their budget and migration constraints and taken as given the policies of the other local government.

Alternatively, we could use regional linear progressive income tax schedules. Adapting the model of Boadway et al (1998) we could assume that households differ in their ability so that for example, immobile households are high ability and mobile workers are low ability.

(iv) Interregional Transfers

The possibility of voluntary interregional transfers needs also to be explored. The question would be to determine whether uncoordinated competition when interregional transfers are feasible yields to the socially optimal
solution. An affirmative answer would imply that there are not efficiency arguments for central government intervention.

For example, Hindriks and Myles (2000) explore the possibility of interregional transfers between regions under decentralized redistribution (tax-transfer competition) and imperfect mobility of two types households, rich and poor. They find out that the result depends on the sequence of the game. If interregional transfers are set simultaneously with redistributive policies they are not sustainable. However, changing the sequence of the game such that regions precommit to interregional transfers before setting their redistributive policy leads to sustainable interregional transfers as a Nash equilibrium. Equilibria with partial or no interregional transfers also emerge. Therefore, we should include interregional (lump sum or matching) transfers in our analysis and explore the different possibilities.

(v) Two Imperfectly Mobile Workers

An interesting extension for further research would be to consider the existence of two types of imperfectly mobile workers: rich and poor. This classification could be linked to ability so that rich households are also high ability type of households and poor households are low ability type. The rent of the fixed factor in each region would be assumed to be shared equally among the residents in the respective region. Alternatively, we could assume that the total rent in the country is shared equally among the total population. Maximizing a welfare function of the kind used in this paper gets a bit cumbersome when having two imperfectly mobile workers. The assumption of symmetric regions would simplify the mathematics considerably. A possibly more interesting alternative which would also simplify the algebra is to change the objective function. The simplest option is to maximize the utility of the poorest individual following the maxi-min social decision rule.

Appendix

A Derivation of Welfare Effects

Differentiating $W_j$ with respect to $z_i$ and using $dW_j = MRS_j dx_j^2 + dx_j^1$ we obtain,

$$
\frac{dW_j}{dz_i} = \left[ (MRS_j - N_j) F_{NN}^j - z_j \right] \frac{\partial N_j}{\partial z_i} \quad (A.1)
$$
Hence inserting into (A.1) the first-order condition (18) for region \( j \) and the migration equilibrium condition derived in (4) gives the horizontal fiscal externality (21). Likewise, when the central government implements a corrective matching grant device the effect on the welfare of region \( i \) from a change in \( z_i \) is defined as,

\[
\frac{dW_i}{dz_i} = (MRS_i - N_i) (1 + F_{NN}^i \frac{\partial N_i}{\partial z_i}) - (1 - s_i)z_i \frac{\partial N_i}{\partial z_i} + s_i N_i = 0 \quad (A.2)
\]

Incorporating \( \frac{\partial N_i}{\partial z_i} \) as defined in (4) and multiplying by \( D \) we obtain the first-order condition (24).

**B Central Government Intervention**

The objective of this section is to derive the required system of matching grants that the central government should implement in order to internalize fiscal externalities. We also show that central government intervention achieves the socially optimal level of overall redistribution.

Incorporating \( \frac{\partial W_i}{\partial z_A} \) and \( \frac{d(\sum_{i=A,B}^j T_i)}{\partial z_A} \) into (27) according to (25) and (26) we obtain,

\[
-\frac{(1-s_B)z_B}{F_{NN}^A-2a} + \frac{F_{NN}^B s_B}{F_{NN}^A-2a} - s_A N_A + s_A z_A \frac{\partial N_A}{\partial z_A} - s_B z_B \frac{\partial N_B}{\partial z_B} = 0 \quad (B.1)
\]

Hence substituting \( \frac{\partial N_i}{\partial z_i} = -\frac{1}{D} \) into (B.1) and rearranging terms yields,

\[
z_j = s_j \left( \frac{z_j}{D} - N_j \right) F_{NN}^j + s_i \left( \frac{z_i}{D} - N_i \right) \left( F_{NN}^i - 2a \right) \quad (B.2)
\]

Following the same steps as we used to derive \( z_i \) from (28),

\[
z_i = s_i \left( \frac{z_i}{D} - N_i \right) F_{NN}^i + s_j \left( \frac{z_j}{D} - N_j \right) \left( F_{NN}^j - 2a \right) \quad (B.3)
\]

Thus,

\[
z_i - z_j = \left[ s_i \left( \frac{z_i}{D} - N_i \right) - s_j \left( \frac{z_j}{D} - N_j \right) \right] 2a \quad (B.4)
\]
Notice that $z_i = z_j$ when $a = 0$. Hence, transfers in each region are identical when workers are perfectly mobile but they differ when there is attachment to regions.

Expressions (B.2) and (B.3) define a system of two equations. Solving this system for $s_i$ and $s_j$ we derive the socially optimal matching grant for each region as defined in (29). Hence, to derive the optimal $s_i$ and $s_j$ in the special case of symmetric regions as defined in (30) just take into account that $F_{NN}^i = F_{NN}^j = F_{NN}, z_i = z_j$ and $N_i = N_j = \frac{N}{2}$.

### B.1 Optimal redistribution with central government intervention

Let us define $\frac{dW_i}{dz_i}$ as,

$$
\frac{dW_j}{dz_i} = \left[(MRS_j - N_j) F_{NN}^j - (1 - s_j) z_j\right] \frac{\partial N_j}{\partial z_i} \quad (B.5)
$$

Then inserting (B.5) and $\frac{d}{dz_i} \sum_{i=A,B} T_i$ into (27) according to (26) yields,

$$(MRS_j - N_j) F_{NN}^j - (1 - s_j) z_j - s_i N_i D + s_i z_i = 0 \quad (B.6)$$

Now incorporating the migration response $\left(\frac{\partial N_i}{\partial z_i}\right)$ into (A.2) we obtain,

$$
\frac{dW_i}{dz_i} = (MRS_i - N_i) \left(\frac{F_{NN}^j - 2a}{D}\right) + (1 - s_i) z_i \frac{1}{D} + s_i N_i = 0 \quad (B.7)
$$

Summing up (B.6) and (B.7) yields,

$$(MRS_i + MRS_j - N) = \frac{2a (MRS_i - N_i) + z_j - z_i}{F_{NN}^j} \quad (B.8)$$

Hence the intervention of the central government achieves the social optimal overall redistribution if and only if (B.8) is equal to zero. This implies,

$$z_i - z_j = 2a (MRS_i - N_i) \quad (B.9)$$
Using (28) and following the same steps we derive a similar condition,

\[ z_i - z_j = 2a (N_j - MRS_j) \]  

(B.10)

Hence, from (B.9) and (B.10) we obtain the necessary condition for a social optimal overall redistribution,

\[ MRS_i + MRS_j = N \]  

(B.11)

as it was to be demonstrated.

C Welfare Impact of New Immigrants

This section is dedicated to analyze the sign of (33), (34) and (35). We also derive conditions (36) and (38).

C.1 Condition (33)

Mathematical manipulation shows that condition (33) can be written as,

\[ a_{jj} (F_{NN}^j - a) - a_{ji} (F_{NN}^i - a) = -a (c_j - d_j N_j) + 1 + \frac{F_{NN}^j}{\theta_i} - a \left( d_j + \frac{\partial (1/\theta_i)}{\partial z_j} D \right) \]  

(C.1)

where \( c_j = \frac{\partial MRS_j}{\partial x^2_j} \leq 0 \) and \( d_j = \frac{\partial MRS_j}{\partial x^1_j} > 0 \) since \( W_j \) is increasing and concave in both \( x^1_j \) and \( x^2_j \). The last term in (C.1), \( \frac{\partial (1/\theta_i)}{\partial z_j} D = \frac{-1}{\theta_i} \left( \frac{\partial F_{NN}}{\partial N_i} \right) \left( \frac{\partial N_i}{\partial z_j} \right) D \leq 0 \) if \( \frac{\partial F_{NN}}{\partial N_i} \geq 0 \) and is zero if the production function, \( F_i \) is quadratic. Hence we assume that \( \frac{\partial F_{NN}}{\partial N_i} < 0 \), zero or if positive small enough so that \( \frac{\partial (1/\theta_i)}{\partial z_j} D \) is compensated by other positive terms in (C.1).

Consequently, if \( z_j > 0 \) it is clear that (C.1) is positive. When \( z_j < 0 \) it is reasonable to consider that the negative terms are dominated by the positive ones such that (C.1) is also positive.
In this subsection we derive the expressions for $\frac{dx_2}{dN}$, $\frac{dx_1}{dN}$ and $dW_i$.

After some mathematical manipulation and expressing the derivatives of $N_i$ with respect to $N$ in terms of derivatives with respect to $z_i$, $\frac{dx_2}{dN}$ as defined in (34) becomes,

$$\frac{dx_2}{dN} = \left[ a_{jj}(F_{NN}^j - a) - a_{ji}(F_{NN}^i - a) \right] \frac{a_{ii}(F_{NN}^i - a) - a_{ij}(F_{NN}^j - a)}{|A|D} \quad (C.2)$$

where $D < 0$ and the terms in square brackets are positive as we showed in the previous subsection.

Then $\frac{dx_1}{dN}$ defined in (35) as $\frac{dx_1}{dN} = -N_i \frac{dx_2}{dN} - z_i \frac{dN_i}{dN}$ can also be written in a similar manner,

$$\frac{dx_1}{dN} = \left[ a_{jj}(F_{NN}^j - a) - a_{ji}(F_{NN}^i - a) \right] \frac{a_{ii}(N_i F_{NN}^i + z_i) - a_{ij}(N_i F_{NN}^j - a) - z_i}{|A|D} \quad (C.3)$$

To infer the sign of (C.3) we need to know the sign of the second term in square brackets. Hence let us write $\left[ a_{ii}(N_i F_{NN}^i + z_i) - a_{ij}(N_i F_{NN}^j - a) - z_i \right]$ in a more convenient way as,

$$\left[ a_{ii}(N_i F_{NN}^i + z_i) - a_{ij}(N_i F_{NN}^j - a) - z_i \right] = c_i z_i + N_i + \frac{N_i F_{NN}^i + z_i}{\theta_j} + \frac{\partial(1/\theta_j)}{\partial z_i} N_i z_i D \quad (C.4)$$

As before $c_i < 0$, $D < 0$ and the sign of $\frac{\partial(1/\theta_j)}{\partial z_i}$ depends on the third-order derivative of the production function with respect to labour. To have a more sharp picture let us drop the last term of (C.4) as if the production function were quadratic. Then the sign of (C.4) depends on the sign of $z_i$.

When $z_i < 0$ (C.4) is clearly positive and $\frac{dx_1}{dN} > 0$. This is different from the result obtained in (Wellisch 1996) where $\frac{dx_1}{dN} > 0$ only when regions are symmetric. However, when $z_i > 0$ we can not infer the sign of (C.4). As a particular case, when regions are symmetric $\frac{dN_i}{dN} = \frac{\partial N_i}{\partial N} = \frac{F_{NN}^i - a}{D} > 0$ and we can directly see from the definition of $\frac{dx_1}{dN} = -N_i \frac{dx_2}{dN} - z_i \frac{dN_i}{dN}$ that the arrival of new immigrants to region $i$ increases the net income of the
immobile factor owner as in (Wellisch, 1996). Hence, we have shown the rational of proposition 2.

Finally, \( \frac{dW_i}{dN} = MRS_i \frac{dx^2_i}{dN} + \frac{dx_i^1}{dN} \) defined in (36) can be expressed as,

\[
\frac{dW_i}{dN} = (MRS_i - N_i) \frac{dx^2_i}{dN} - z_i \frac{dN_i}{dN} \tag{C.5}
\]

Then using the first-order condition (18) incorporating \( \frac{dx^2_i}{dN} \) and \( \frac{dN_i}{dN} \) and after some mathematical manipulation (C.5) becomes,

\[
\frac{dW_i}{dN} = -z_i a_{ii} \left[ a_{jj} (F_{NN}^j - a) - a_{ji} (F_{NN}^i - a) \right]
\]

\[
\left| A \right| (F_{NN}^j - 2a)
\]

as defined in (37).

**C.3 \( \frac{dW}{dN} \) with central government intervention**

From \( \frac{dW_i}{dN} = MRS_i \frac{dx^2_i}{dN} + \frac{d(F_i - N_i F_k^i)}{dN} - \frac{d(1-s_i) N_i z_i}{dN} - \frac{dT_i}{dN} \) and taking into account (22) we derive,

\[
\frac{dW_A}{dN} + \frac{dW_B}{dN} = \sum_{i=A,B} MRS_i \frac{dx^2_i}{dN} + \sum_{i=A,B} \frac{d(F_i - N_i F_k^i)}{dN} - \sum_{i=A,B} \frac{d(N_i z_i)}{dN}
\]

\[
= \sum_{i=A,B} (MRS_i - N_i) \frac{dx^2_i}{dN} + \sum_{i=A,B} N_i \frac{d(F_k^i + z_i)}{dN} + \sum_{i=A,B} \frac{d(F_i - N_i F_k^i)}{dN} - \sum_{i=A,B} \frac{d(N_i z_i)}{dN}
\]

\[
= \sum_{i=A,B} (MRS_i - N_i) \frac{dx^2_i}{dN} - z_i \sum_{i=A,B} \frac{dN_i}{dN} \tag{C.7}
\]

as defined in (38).
References


