Controlling Pollution with Relaxed Regulations*

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Abstract

In this paper, we investigate the features of optimal environmental policies composed of pollution standards and costly inspection processes, where fines for exceeding the standards depend both on the degree of transgression and the environmental technology that the firm uses in its production process. We show that the main characteristics of these policies depend crucially on when the firm selects that technology with respect to the timing of the policy announcement. In fact, we find that the firm has incentives to *over-invest* in *green* technologies when the policy is announced afterwards; and to *under-invest* in them if the environmental authority plays first. Surprisingly, we find that both the firm and the regulator prefer that the firm anticipates the technology investment to the policy announcement, even when this implies that expected penalties for noncompliance with the environmental standards might be zero.

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1 Introduction

The purpose of environmental regulations is to protect individuals against pollution. These regulations are diverse and currently used by governments worldwide. For instance, authorities require polluting firms to comply with prescribed pollution limits or *standards*, and persuade them to use *clean*, though expensive, production processes. Normally, regulators cannot implement these policies easily, since they do not observe without cost the performance of firms with respect to the policies. Therefore, they also design inspection programs that consist of monitoring frequencies and sanctions in case firms are found to be violating the policies. Sometimes, it is argued that these regulations are soft, in the sense that either inspections are very infrequent or that sanctions are very small.[1]

In this paper, we analyze the rationale for the leniency of the regulations from a different perspective of that considered in the literature. We present a principal-agent model in which the regulator chooses the terms of the policy − the standards and the probabilities of inspection − considering that fines for noncompliance depend on two factors: the degree of violation, that is, the difference between the observed pollution level and the standard[2]; and the investment in pollution control technology, in a way such that the more expenditure in technology, the smaller the fine. To our knowledge, this second aspect has not been studied before, in particular its effect on the pollution level and the environmental technology to be employed in the production process, both of them decided by the polluting firm.

We demostrate that a key factor to explain why regulations might be lenient is the timing at which its respective decision is made, in particular the investment in the pollution technology. In some instances, the firm may decide to anticipate this investment as a means of preventing from future more rigid environmental policies. In other cases, the firm may decide to invest

[1]The fact that monitoring is expensive is the most common reason given for that infrequency of inspections. Among the reasons that can explain the leniency of the fines are those of Polinski and Shavell (1979), who assume that agents are risk averse; Bebchuk and Kaplow (1991) and Kaplow (1990), who consider that agents have imperfect information about the regulatory policy; Polinski and Shavell (1990), who study the case of agents' differences in wealth; Shavell (1992), who considers the case of marginal deterrence; among others.

[2]To be precise, we consider that fines are increasing and convex in the degree of violation. This progressiveness of the sanction is not sustained on the grounds of moral considerations only but also on efficiency grounds. See Shavell (1992) for a discussion on this issue.

in technology just to adapt to the latest environmental requirements. We find that the results dramatically differ depending on when the investment is made with respect to the policy announcement.

When the firm anticipates the investment in technology to the regulatory announcement, we find that the firm has incentives to over-invest in technology if the fines for noncompliance are contingent on that technology. This is so because the additional expenditure in a cleaner technology is compensated by the savings in the expected fines for noncompliance: on the one hand, the cleaner the technology, the smaller the penalty itself; on the other hand, the overinvestment in technology serves as a signal for the regulator, that might decide not to inspect the firm to measure the degree of noncompliance with the standards. We show that in this case the firm prefers that fines are contingent on the technology, while the regulator prefers that fines depend on the degree of noncompliance only.

However, when the firm decides the investment in technology after the policy announcement, the firm under-invests in the pollution technology. In this case, the environmental policies are more rigid than in the previous case, since the firm cannot send any signal prior to the establishment of the regulation. Here, the firm also prefers that fines depend on the technology investment. However, and contrary to the previous case, the regulator may prefer that fines are contingent on the technology also. If fines do not depend on that investment, the firm may decide not to invest in environmental technology at all since it cannot affect the policy, and therefore, environmental damages can be very large.

Finally, we find that it is benefitial for both the firm and the regulator that the firm anticipates the investment in technology to the policy announcement, even when the firm prefers fines contingent on the technology investment and the regulator does not. The result is surprising from the point of view of the regulator, since the induced regulation in this case is more lenient, i.e., expected fines are smaller. However, we show that the firm induces that leniency in the regulation with the overinvestment in technology, that results in a savings in environmental damages that benefits social welfare.

The enforcement aspect of environmental regulation has been widely studied (see Heyes (2000) and the references cited therein for a complete review). All this literature starts with Becker (1968) in the crime context where, considering a fixed standard, the issue is the joint endogeneization of fines and probabilities of inspection.[3] Moving from the basis, there are three strands of thought in the environmental context that have received special atention, such as contested enforcement (that is, the case in which agents can take actions to evade penalties in case they are discovered to be violating regulations, such as in Kambhu (1989) or Malik (1990)); self-reporting (i.e., when agents can declare their respective status to an enforcement agency, such as in Malik (1993) or in Livernios and Mackena (1999) and, more generally, in the context of tax evasion[4]) and multi-period enforcement (when the agents and the environmental authority interact through time, being the regulations contingent on previous actions taken by the agents, such as in Harrington (1988) and Harford and Harrington (1992)).

Our paper does not fit in any of the three strands mentioned above. Instead, our main contribution is to generalize the basic regulatory model as a means of jointly obtaining the optimal pollution standards, probabilities of inspection and fines for noncompliance, a problem that has not been solved before.[5]

The remainder of the paper is organized as follows. In the next Section, we present the model. In Section 3, we study the case in which the firm anticipates the technology investment to the policy announcement. In Section 4, we analyze the case in which the firm waits until the policy is announced. In Section 5, we compare the results of the two previous Sections in terms of the firm's expected payoff and the social welfare. Finally, we conclude in Section 6.

2 The Model

We consider a firm that emits pollution $e > 0$ as a result of its production process. The relationship between production and pollution is monotone, that is, more production is associated with more pollution, and *vice versa*. Also, the amount of pollution is related to the type of technology that the

[3] Becker (1968) was the first who proved the optimality of imposing maximal fines, given costly enforcement. Subsequently, also in the crime context, there have been several works explaining the reasons why, in fact, fines are not maximal (see footnote 1)

[4] In this context, there are few papers that consider the endogenization of fines. Some examples include Mookherjee and Png (1989) and Pestieu et al. (1997), who assume that fines are constant, that is, independent of the degree of violation; and Greenberg (1984), in the context of multi-period enforcement.

[5] In Arguedas (1999), this problem is partially solved in a context of bargaining, that is, in a case in which firms and the environmental authority can negotiate the stringency of the penalties.

firm uses, $\beta \in \left[0, \overline{\beta}\right]$, where $\overline{\beta} > 1$. That is, given a production level, the *dirtier* the technology (represented by a larger β), the larger the pollution level. The firm obtains private profits that depend on the pollution level and the type of technology that the firm employs, represented by the function:

$$b(e, \beta) = ke - \frac{e^2}{\beta} \tag{1}$$

where $k > 0$ represents the degree of *profitability* of the firm. Note that, given β, profits are strictly concave in the pollution level with an interior maximum at $e = \frac{k\beta}{2}$. Also, profits are strictly increasing in β, meaning that dirtier technologies are cheaper and, therefore, profits associated with these technologies are larger.

Pollution generates external damages that depend on the pollution level and the dirtiness of the technology also, as follows:

$$d(e, \beta) = \beta e^2 \tag{2}$$

Then, for a given technology, damages are increasing and convex in the pollution level. Also, the dirtier the technology, the larger the associated damages of a given pollution level, and *vice versa*.

In the absence of a regulation, the firm does not internalize the presence of external damages and it selects the technology and the pollution level that maximize (1), that is, $\beta = \overline{\beta}$ and $e = \frac{k\overline{\beta}}{2}$, obtaining private profits of $\frac{k^2\overline{\beta}}{4}$. By contrast, if damages are fully internalized (i.e., if $(1) - (2)$ is maximized), we obtain $\beta^* = 1$ and $e^* = \frac{k}{4}$, i.e., the *efficient* levels, which correspond to private profits of $\frac{3k^2}{16}$, smaller than in the previous case.

We assume that there exists an environmental authority (a planner) that is concerned about the above external damages and regulates the polluting activity. To do so, the planner sets a standard $s \geq 0$, that is, a maximum level of permitted pollution. We consider that the planner observes the technology that the firm uses but cannot know the emitted pollution level unless it monitors the firm, which costs $c > 0$. This means that the planner needs to inspect the firm to verify its performance with respect to the standard. Since monitoring is costly, though perfectly accurate, it is not necessarily optimal to monitor the firm always, but randomly. Therefore, the planner also sets the probability of inspection, $p \in [0, 1]$.

If, once monitored, the firm is discovered to be exceeding the standard, then it is forced to pay a sanction that depends both on the degree of violation, $e - s$, and the type of technology that the firm uses, β. We assume that the sanction is not decided by the planner, but is given exogenously by a government entity other than the planner, for example, the legislature. The penalty function takes the following structure:

$$f(\beta, e-s) = \begin{cases} \phi(\beta)(e-s)^2 & e-s>0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where $\phi(\beta) \geq \beta$ and $\phi'(\beta) \geq 0$. Note that, given β, the fine is increasing and convex in the degree of violation, that is, the penalty is contingent on the magnitude of the crime. Also, given a degree of transgression $e-s$, the penalty is nondecreasing in the dirtiness of the technology, which can be interpreted as a *reward* for investing in cleaner, more expensive technologies.[6] Throughout the paper, we study the effects of considering two alternatives for the dependency of the fines on β: $\phi(\beta) = \beta$ and $\phi(\beta) = t$. In particular, we analyze what structure is better from both the regulator's and the firm's view points, under alternative assumptions.

We consider a principal-agent framework in which the planner chooses the policy instruments (the standard s and the probability of inspection p) considering that sanctions for noncompliance are given and anticipating the firm's optimal response to the policy. The firm selects the pollution level and the technology to maximize its expected payoff, which includes not only its private profits (given by (1)) but also the expected fine in case it decides to violate the standard. Formally, the firm's expected payoff function is the following:

$$\Pi(e, \beta, s, p) = ke - \frac{e^2}{\beta} - pf(\beta, e-s) \quad (4)$$

To derive the optimal policy, we assume that the planner maximizes a expected social welfare function that considers the firm's expected payoff (given by (4)), the external damages, the expected collection of the fines and the expected monitoring costs. Considering that imposing fines is socially costless, the expected social welfare function can be written as follows:

$$R(e, \beta, s, p) = ke - \frac{e^2}{\beta} - \beta e^2 - pc \quad (5)$$

As described in the Introduction, we consider two possibilities depending on the timing of the technology investment decision with respect to the policy announcement. If the firm anticipates the investment decision, we consider a three-stage game in which the firm first chooses the pollution technology,

[6] We consider $\phi(\beta) \geq \beta$ to ensure that $p \leq 1$ later on in Lemma 2 and Proposition 3.

β; considering that technology, the planner then announces both the standard and the probability of inspection, that is, the policy (s, p); and, finally, considering that policy and the previously selected technology, the firm decides the pollution level that maximizes its expected payoff. By contrast, if the firm waits until the policy is announced, we have a two-stage game in which the regulator first announces the policy elements, (s, p), and the firm then selects both the pollution level and the technology that maximizes its expected payoff, considering the previously announced policy.

In the following section, we analyze the characteristics of the optimal policy when the firm anticipates the investment decision to the policy announcement, that is, the three-stage game.

3 Results of the three - stage game

The timing of the problem is the following. First, the firm chooses the technology β; then, the planner selects the policy (s, p); and, finally, the firm decides the pollution level e. We solve the problem by backward induction to find the subgame perfect equilibrium. That is, in the first stage we obtain the firm's optimal pollution level, given the policy and the pollution technology. In the second stage, we find the optimal policy, given the technology and considering the optimal pollution level obtained in stage one. Finally, in the third stage we obtain the optimal technology considering both the optimal pollution level and the optimal policy obtained in the two previous stages.

3.1 Stage 1

Given $\beta \in \left[0, \overline{\beta}\right]$ and the policy $s \geq 0$, $p \in [0, 1]$, the firm solves the following problem:

$$\begin{aligned} Max_e \qquad & \left\{ ke - \frac{e^2}{\beta} - p\phi\left(\beta\right)\left(e - s\right)^2 \right\} \\ s.a. \qquad & s - e \leq 0 \\ & e - \frac{k\beta}{2} \leq 0 \end{aligned} \tag{6}$$

The first restriction of the above problem guarantees that the firm chooses a pollution level that is, at least, as large as the prescribed standard (else, considering the fine in the objective function has no sense). The second restriction ensures that the pollution level is at most the one the firm would choose in the absence of the regulation. These two restrictions combined guarantee that the firm chooses the pollution level on the basis of a meaningful regulation.

The Lagrangian of problem (6) is the following:

$$L(e, \lambda_1, \lambda_2) = ke - \frac{e^2}{\beta} - p\phi(\beta)(e-s)^2 - \lambda_1 (s-e) - \lambda_2 \left(e - \frac{k\beta}{2}\right) \quad (7)$$

where (λ_1, λ_2) are the corresponding Lagrange multipliers. The necessary optimality conditions are the following:[7]

$$\begin{aligned}
&\frac{\partial L}{\partial e} = k - \frac{2e}{\beta} - 2p\phi(\beta)(e-s) + \lambda_1 - \lambda_2 = 0 \\
&\lambda_1 (e-s) = 0 \\
&\lambda_2 \left(e - \frac{k\beta}{2}\right) = 0 \\
&s - e \leq 0; \; e - \frac{k\beta}{2} \leq 0; \quad \lambda_1 \geq 0; \quad \lambda_2 \geq 0
\end{aligned}$$

These conditions lead to the following:

Lemma 1 *The solution to problem* (6) *is:*

$$e = \frac{\beta(k + 2ps\phi(\beta))}{2(1 + p\beta\phi(\beta))}; \; \lambda_1 = \lambda_2 = 0; \quad \text{where } s \leq \frac{\beta k}{2} \quad (8)$$

■

Observe that, as expected, the pollution level is positively related to the standard and negatively related to the probability of inspection.[8] As far as the effect of the technology is concerned, we have that only when either there is no regulation (i.e., $p = 0$) or if the fines do not depend on the technology (i.e., $\phi' = 0$), there exists a clear monotone relationship between the pollution level and the technology, that is, the dirtier the technology, the larger the associated pollution level, and vice versa. However, there are other instances in which that is not the case, with a crutial effect on the results, as we will see later on.

[7]Note that these conditions are also sufficient, since (6) is a concave program.

[8]In the particular case that $s = \frac{\beta k}{2}$, we then have $e = s = \frac{\beta k}{2}$. Note that, in this case, the regulator announces the same pollution level that the firm would select in the absence of regulation.

3.2 Stage 2

We now present the regulator's problem considering the optimal response of the firm, given in (8).

$$\begin{aligned} &Max_{s,p} && \left\{ke - \frac{e^2}{\beta} - \beta e^2 - pc\right\} \\ &s.a && s - \frac{\beta k}{2} \leq 0 \\ & && -s \leq 0 \\ & && -\ p \leq 0 \\ & && p - 1 \leq 0 \end{aligned} \tag{9}$$

Now, the Lagrangian of the problem is:

$$L\left(s, p, \mu_1, \mu_2, \mu_3, \mu_4\right) = ke - \frac{e^2}{\beta} - \beta e^2 - pc - \mu_1\left(s - \frac{\beta k}{2}\right) + \mu_2 s + \mu_3 p - \mu_4\left(p - 1\right)$$

where $(\mu_1, \mu_2, \mu_3, \mu_4)$ are the Lagrange multipliers and $e = \frac{\beta(k+2ps\phi(\beta))}{2(1+p\beta\phi(\beta))}$. We present the solution in the following:[9]

Lemma 2 *The solution to problem (9) is given by the following conditions:*

i) If $0 \leq c \leq \frac{\beta^4\phi(\beta)k^2}{2}$, *then*

$$\begin{aligned} &s = 0 \\ &g\left(\beta, p\right) = \beta^3 k^2 \phi\left(\beta\right)\left(\beta - p\phi\left(\beta\right)\right) - 2c\left(1 + p\beta\phi\left(\beta\right)\right)^3 = 0 \\ &\beta - p\phi\left(\beta\right) \geq 0 \\ &\mu_1 = \mu_3 = \mu_4 = 0;\ \mu_2 = \frac{2cp\left(1 + p\beta\phi\left(\beta\right)\right)}{k\beta} \geq 0 \end{aligned} \tag{10}$$

ii) If $c \geq \frac{\beta^4\phi(\beta)k^2}{2}$, *then*

$$s = 0;\ p = 0;\ \mu_1 = \mu_2 = \mu_4 = 0;\ \mu_3 = c - \frac{\beta^4\phi\left(\beta\right)k^2}{2} \geq 0 \tag{11}$$

[9] From now on, we omit to write the Kuhn-Tucker conditions associated to each optimization program, since they are constructed in a similar way to those presented for problem (6). Also, we have checked that second order conditions are satisfied in all the problems.

■

Note that, given an inspection cost $c > 0$, it is more likely that the regulator inspects the firm when either the technology is dirty enough (i.e., when β is large) or if the firm's degree of profitability (k) is large enough.

On the one hand, if monitoring is not expensive (case i)), then the optimal policy is such that the standard is zero and the probability of inspection is given by the implicit expression $g(\beta, p) = 0$. If the regulator considered an infinitesimal increase in the standard, social welfare would decrese by μ_2, the Lagrange multiplier. This is due to the fact that the regulator should have to increase the inspection probability as well to induce the firm to pollute the same amount, which would increase monitoring costs. Also, as expected, the optimal probability of inspection decreases when monitoring costs increase and when the degree of firm's profitability decrease. Considering (8) and (10), the optimal pollution level reduces to:

$$e(\beta, p) = \frac{\beta k}{2(1 + p\beta\phi(\beta))} \tag{12}$$

On the other hand, if monitoring is expensive enough (case ii)), the best is to leave the firm unregulated (again, $\mu_3 \geq 0$ shows that an infinitesimal increase in the probability of inspection would reduce social welfare) and then the level of the standard loses importance (reflected in $\mu_2 = 0$). In this case, considering (8) and (11), the firm chooses $e = \frac{\beta k}{2}$.

Therefore, observe that, regardless of the levels of the monitoring cost or the technology investment, we always obtain that the optimal standard is zero, that is, we always obtain a corner solution.[10]

3.3 Stage 3

Finally, we find the technology that maximizes the firm's expected payoff considering the solution to the two previous Lemmas. First, considering (10) and (12), the firm's expected payoff reduces to the following expression:

$$\Pi(\beta) = ke - \frac{e^2}{\beta} - p\phi(\beta)(e - s)^2 = \frac{ek}{2} \tag{13}$$

[10] We obtain an interior solution only when $c = 0$, which is equivalent to the one presented in Lemma 2, as we will see later on in stage three. The solution is $s = \frac{k\beta(p\phi(\beta) - \beta)}{2p\phi(\beta)(1+\beta^2)}$ and $\frac{\beta}{\phi(\beta)} \leq p \leq 1$, that results in $e = \frac{k\beta}{2(1+\beta^2)}$.

That is, the firm's expected payoff is an increasing linear function of the pollution level. We now present the problem the firm faces in this stage:[11]

$$
\begin{aligned}
Max_{\beta,p} \quad & \frac{ek}{2} \\
s.a. \quad & \beta - \overline{\beta} \leq 0 \\
& g(\beta, p) = k^2 \beta^3 \phi(\beta - p\phi) - 2c[1 + p\beta\phi]^3 = 0 \\
- \quad & \beta + p\phi(\beta) \leq 0 \\
& -p \leq 0 \\
& p - 1 \leq 0
\end{aligned} \tag{14}
$$

The first restriction stands for the admissible range of choices of the technology β. Restrictions two to five come from stage two above, and link the technology and the probability of inspection for given parameters k and c. Note that those restrictions come specifically from part i) of Lemma 2, where $0 \leq c \leq \frac{\beta^4 \phi(\beta) k^2}{2}$, which corresponds to a positive probability of inspection. Remember that part ii) of Lemma 2 summarizes the case of no regulation, and the solution to stage three in that case is obvious, i.e., $\beta = \overline{\beta}$.

The Lagrangian of problem (14) is the following:

$$
\begin{aligned}
L(\beta, p, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5) \quad = \quad & \frac{ek}{2} - \gamma_1 \left(\beta - \overline{\beta}\right) - \gamma_2 g(\beta, p) \\
& -\gamma_3 (p\phi(\beta) - \beta) + \gamma_4 p - \gamma_5 (p - 1)
\end{aligned}
$$

where $\left(\gamma_{1,}..., \gamma_5\right)$ are the corresponding Lagrange multipliers. In the following Proposition, we present the solution.[12]

Proposition 3 *Given a monitoring cost $c > 0$ and a degree of firm's profitability $k > 0$, if the firm chooses the technology before the policy announcement and fines for noncompliance depend monotonically on β (i.e.,*

[11] To be precise, in this stage the firm decides the level of the technology only, but it induces the probability of inspection through the expression $g(\beta, p) = 0$ obtained in stage two, due to the backward induction process that we are considering. Therefore, the problem in which the firm decides the level of the technology only is mathematically equivalent to the problem in which the firm decides both the technology and the probability of inspection. Here, we analyze the latter for purposes of clarity in presenting the results, and also to stress the fact that in this case the firm has certain power to manipulate the regulatory policy in its own benefit.

[12] The necessary optimality conditions for problem (14) and the relevant expressions we have used in finding the solution are in the Appendix.

$\phi'(\beta) \geq 0$), then the solution of problem (14) is given by the following conditions:

i) If $k^2\beta^4(4\phi+\beta\phi')-k^2\beta^2\phi-6c \leq 0$, then:

$$
\begin{aligned}
&\beta^2\phi'(p\phi-\beta)+\phi\left(1-\beta^2\right)=0\\
&k^2\beta^3\phi(\beta-p\phi)-2c\left[1+p\beta\phi\right]^3=0\\
&\gamma_1=\gamma_3=\gamma_4=\gamma_5=0\\
&\gamma_2=\frac{k^2\beta}{4(1+p\beta\phi)^2\left[k^2\beta^2\phi+6c(1+p\beta\phi)^2\right]}>0
\end{aligned}
$$

ii) If $k^2\beta^4(4\phi+\beta\phi')-k^2\beta^2\phi-6c \geq 0$, then:

$$
\begin{aligned}
&k^2\beta^4\phi-2c=0;\ p=0\\
&\gamma_1=\gamma_3=\gamma_5=0\\
&\gamma_2=\frac{1}{4\beta^3\left[4\phi+\beta\phi'\right]}>0\\
&\gamma_4=\frac{k^2\beta^5\phi(4\phi+\beta\phi')-\beta\phi\left(k^2\beta^2\phi+6c\right)}{4\beta^3(4\phi+\beta\phi')}\geq 0
\end{aligned}
$$

■

First note that we find either an interior solution (case i) or a corner solution (case ii) depending on the parameters of the problem. The condition to have either one solution or the other depends on β also, which refers to the optimal investment in technology found in each case. Later on, this condition becomes more clear considering the two particular cases, $\phi(\beta)=\beta$ and $\phi(\beta)=t$.

Concentrating on case i) of the Proposition, and since $p\phi-\beta \leq 0$, we obtain that the optimal technology investment is at most 1. Remember that the efficient technology level is $\beta^*=1$ (see Section 2, above). This means that the firm over-invests in a clean technology with respect to the efficient level as a means of having a more favourable policy with a smaller expected penalty for noncompliance. Note that in this case the optimal probability is positive, and it decreases as the monitoring costs increase.

In case ii) of the Proposition, we obtain that the optimal probability of inspection is zero. In this case, the firm induces the non-regulation with a sufficiently large effort in technology investment, as we will see later on in the examples. Observe that this result is different from the one in which the optimal probability of inspection is zero because monitoring costs are large enough (case ii of Lemma 2). Remember that we are constraining the resolution of stage three to case i) of Lemma 2.

In order to explain the overall solution of the problem, we now simplify the penalty structure considering two alternatives for $\phi(\beta)$. In Corollary 4, we summarize the solution of the case in which $\phi(\beta) = \beta$. In Corollary 5, we present the case in which $\phi(\beta) = t$. In both cases, we have added (as case iii) the possibility that $c \geq \frac{\beta^4 \phi(\beta) k^2}{2}$, that corresponds to part ii) of Lemma 2, in which there is no regulation and, therefore, $e = \frac{\overline{\beta} k}{2}$ and $\beta = \overline{\beta}$ (see Section 2, above).

Corollary 4 *Given a monitoring cost $c > 0$ and a degree of firm's profitability $k > 0$, if the firm chooses the technology before the policy announcement and fines for noncompliance are such that $\phi(\beta) = \beta$, then the overall solution to the problem is given by the following conditions:*

i) If $0 \leq c \leq \frac{k^2}{2^{7/2}}$, then:

$$p = 2 - \frac{1}{\beta^2}$$
$$k^2 \left(1 - \beta^2\right) - 16c\beta^3 = 0$$
$$s = 0; \;\; e = \frac{k}{4\beta}$$

ii) If $\frac{k^2}{2^{7/2}} \leq c \leq \frac{k^2 \overline{\beta}^5}{2}$, then:

$$p = 0; \;\; \beta = \left(\frac{2c}{k^2}\right)^{1/5}; \;\; s \geq 0; \;\; e = \left(\frac{k^3 c}{16}\right)^{1/5}$$

iii) If $c \geq \frac{k^2 \overline{\beta}^5}{2}$, then:

$$p = 0; \;\; \beta = \overline{\beta}; \;\; s \geq 0; \;\; e = \frac{\overline{\beta} k}{2}$$

■

In Figure 1, we represent the solution, specifically the relationships between the optimal technology β and the monitoring cost c, and the optimal pollution level e and the monitoring cost. In the horizontal axis of both graphs, we measure the monitoring cost. In the vertical axis of each graph, we measure the technology and the pollution level, respectively.

When monitoring is cheap enough (case i), we have that it is optimal for the regulator to monitor the firm (that is, $p > 0$), and the smaller the cost, the larger the probability of inspection. Observe that, in the particular case in which $c = 0$, we have that $p^* = 1$, $\beta^* = 1$ and $e^* = \frac{k}{4}$. That is, the regulator can induce the efficient solution when monitoring is costless only, with a hundred percent chance of inspection. Also, it is easy to verify that there exists a negative relationship between the technology level and the monitoring cost, and a positive relationship between the pollution level and the monitoring cost. Therefore, in this case the firm over-invests in technology with respect to the efficient level.

When monitoring is expensive enough (cases ii and iii), it is optimal to leave the firm unregulated (that is, $p = 0$). However, there is a range of values of the monitoring cost (case ii) for which the firm is inducing $p = 0$ through the appropriate selection of the technology and the pollution level. The larger the monitoring cost, the larger both the technology and the pollution level necessary to induce a zero inspection probability. Finally, there exists a threshold value of the monitoring cost ($c = \frac{k^2\overline{\beta}^5}{2}$) beyond which $\beta = \overline{\beta}$ and $e = \frac{\overline{\beta}k}{2}$, respectively, the technology and pollution level chosen by the firm in the absence of a regulation.

Next, we present the solution of the case in which $\phi(\beta) = t$.

Corollary 5 *Given a monitoring cost $c > 0$ and a degree of firm's profitability $k > 0$, if the firm chooses the technology before the policy announcement and fines for noncompliance are such that $\phi(\beta) = t \geq 1$, then the overall solution to the problem is given by the following conditions:*

i) If $0 \leq c \leq \frac{k^2 t}{2}$, then:

$$k^2 t (1 - pt) - 2c (1 + pt)^3 = 0$$
$$\beta = 1; \ s = 0; \ e = \frac{k}{2(1 + pt)}$$

ii) If $\frac{k^2 t}{2} \leq c \leq \frac{k^2 t \overline{\beta}^4}{2}$, then:

$$p = 0; \ \beta = \left(\frac{2c}{k^2 t}\right)^{1/4}; \ s \geq 0; \ e = \left(\frac{ck^2}{8t}\right)^{1/4}$$

iii) If $c \geq \frac{k^2 t \overline{\beta}^4}{2}$*, then:*

$$p = 0;\ \beta = \overline{\beta};\ s \geq 0;\ e = \frac{\overline{\beta}k}{2}$$

■

In Figure 2, we represent the solution, again the relationships between the monitoring cost and the technology level, and the monitoring cost and the pollution level. Here, when monitoring is cheap enough (case i), the firm is monitored, but the fine for noncompliance does not depend on the technology β. In this case, the optimal technology level is fixed on that range and equal to 1, i.e., the efficient level. Here, in the particular case in which $c = 0$, we have that $p^* = \frac{1}{t}$, $\beta^* = 1$ and $e^* = \frac{k}{4}$. That is, the regulator can induce the efficient solution when monitoring is costless only, as in the previous case. The difference is that here the regulator needs to inspect the firm with a frequency $\frac{1}{t}$, smaller than in the previous case and, therefore, with smaller associated monitoring costs.

When monitoring is expensive enough (cases ii and iii), the firm is leaved unregulated. In case ii), as in the previous Corollary, the firm induces that non-regulation through the appropriate selection of β.

An interesting question to ask is the scheme preferred by the regulated firm. First, we observe that in both cases private benefits increase as the monitoring cost increase, since, by (13), there exists a positive relationship between the pollution level and private profits. Considering that expression, we have that private profits are equal to $\frac{k^2}{8}$ when $c = 0$ in both cases. Also, the firm obtains private profits equal to $\frac{k^2}{4}$ in both cases when either $c = \frac{k^2}{2}$ and $\phi(\beta) = \beta$ or when $c = \frac{k^2 t}{2}$ and $\phi(\beta) = t$. Since we are assuming that $t \geq 1$, we have that the firm prefers a fine contingent on β to a fixed fine, at least when the regulator imposes a positive probability of inspection. This seems logical, since the firm has more flexibility to affect the fine when it is contingent on β. However, when monitoring costs are large enough, we might have the opposite result if $\overline{\beta} > t$, since the firm may achieve maximum profits $\frac{k^2 \overline{\beta}}{4}$ earlier when fines do not depend on β.

And, which is the scheme preferred by the regulator? First, it is clear that social welfare decreases when monitoring costs increase. Considering (5), social welfare is $\frac{k^2}{8}$ when $c = 0$, equal in both cases and the maximum that can be obtained. Also, social welfare is 0 when either $c = \frac{k^2}{2}$ and $\phi(\beta) = \beta$ or when $c = \frac{k^2 t}{2}$ and $\phi(\beta) = t$. This means that, at least for the range of values of the monitoring costs for which the regulator imposes a positive probability of inspection, the regulator prefers the scheme in which $\phi(\beta) = t$. However, as before, if monitoring costs are large enough and $\overline{\beta} > t$, social welfare achieves its minimum earlier under $\phi(\beta) = t$ than under $\phi(\beta) = \beta$, which means that, at least for a range of values, the regulator might prefer the scheme $\phi(\beta) = \beta$.

4 Results of the two - stage game

Now, the regulator first announces the policy (s, p). The firm then selects both the pollution level e and the technology β. As in the previous Section, we solve the problem backwards to find the subgame perfect equilibrium. Therefore, in the first stage we find the optimal firm's response to any announced regulatory policy. Then, in the second stage we find the policy that maximizes social welfare considering the previously obtained optimal firm's response.

In order to establish a comparison with the results of the previous Section, we analyze the two cases, $\phi(\beta) = \beta$ and $\phi(\beta) = t$. Now, we consider the resolution of each case separately from the first stage, since the specific form of $\phi(\beta)$ affects the firm's decision on β and e.

4.1 First stage when $\phi(\beta) = \beta$

Given the policy (s, p), the firm solves the following problem[13]:

$$\begin{aligned} & Max_{e,\beta} && \left\{ke - \frac{e^2}{\beta} - p\beta\,(e-s)^2\right\} \\ & \quad s.a. && s - e \leq 0 \\ &&& e - \frac{k\beta}{2} \leq 0 \qquad (15) \\ &&& \beta - \overline{\beta} \leq 0 \qquad (16) \end{aligned}$$

The Lagrangian of this problem is

$$L(e, \beta, \lambda_1, \lambda_2, \lambda_3) = ke - \frac{e^2}{\beta} - p\beta\,(e-s)^2 - \lambda_1 (s-e) - \lambda_2 \left(e - \frac{k\beta}{2}\right) - \lambda_3 \left(\beta - \overline{\beta}\right)$$

and the solution is given in the following:

[13]The justification of the restrictions of this problem is the same as for problem (6).

Lemma 6 *The solution to problem* (15) *is the following:*

i) If $\left(k+2p^{1/2}s\right)-\overline{\beta}p^{1/2}\left(k-2p^{1/2}s\right)\leq 0$, *then:*

$$\begin{aligned} e &= \frac{k+2p^{1/2}s}{4p^{1/2}};\ \beta=\frac{k+2p^{1/2}s}{kp^{1/2}-2ps} \\ \lambda_1 &= \lambda_2=0;\ k>2p^{1/2}s \end{aligned} \tag{17}$$

ii) Else:

$$\begin{aligned} e &= \frac{\overline{\beta}\left(k+2p\overline{\beta}s\right)}{2\left(1+p\overline{\beta}^2\right)};\ \beta=\overline{\beta} \\ \lambda_1 &= \lambda_2=0;\ \lambda_3=\frac{e^2}{\beta^2}-p\left(e-s\right)^2\geq 0 \end{aligned}$$

■

In case i), as expected, the pollution level is positively related to the standard and negatively related to the probability of inspection. Also, the technology level is positively related to the standard, and negatively related to the probability of inspection only when we have $s=0$. In case ii), the sense of the relationships of the pollution level and both the standard and the probability of inspection are the same as those of case i). Note that in this second case, we obtain exactly the same solution as the one presented in Lemma 1 when $\phi\left(\beta\right)=\beta$ and $\beta=\overline{\beta}$.

Therefore, in the second stage of the problem, we concentrate in case i) of Lemma 6, since the solution to case ii) is exactly the same to the one presented in Lemma 2, when $\beta=\overline{\beta}$. However, in Proposition 7 we consider the overall solution and, therefore, cases ii) and iii) of the Proposition are equivalent to cases i) and ii) of Lemma 2 when $\beta=\overline{\beta}$.

4.2 Second stage when $\phi\left(\beta\right)=\beta$

The regulator's problem, considering Lemma 6, is the following:

$$\begin{aligned} & Max_{s,p} && \left\{ke-\frac{e^2}{\beta}-\beta e^2-pc\right\} \\ & s.a. && \left(k+2p^{1/2}s\right)-\overline{\beta}p^{1/2}\left(k-2p^{1/2}s\right)\leq 0 \\ & && 2p^{1/2}s-k\leq 0 \end{aligned}$$

$$
\begin{aligned}
- \quad & s \leq 0 \\
- \quad & p \leq 0 \\
& p - 1 \leq 0
\end{aligned}
$$

Now, the Lagrangian is

$$
\begin{aligned}
L\left(s, p, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5\right) = \quad & ke - \frac{e^2}{\beta} - \beta e^2 - pc - \mu_1 \left[\left(k + 2p^{1/2}s\right) - \overline{\beta}p^{1/2}\left(k - 2p^{1/2}s\right)\right] \\
& -\mu_2\left(2p^{1/2}s - k\right) + \mu_3 s + \mu_4 p - \mu_5 (p-1)
\end{aligned}
$$

where $\left(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\right)$ are now the corresponding Lagrange multipliers.

The overall solution to the problem is presented in the following:

Proposition 7 *Given a monitoring cost $c > 0$ and a degree of firm's profitability $k > 0$, if the firm chooses the technology after the policy announcement and fines for noncompliance are such that $\phi(\beta) = \beta$, then the overall solution to the problem is given by the following conditions:*

i) If $0 \leq c \leq \frac{k^2\overline{\beta}^3\left(\overline{\beta}^2 - 1\right)}{16}$, *then*

$$
\begin{aligned}
& s = 0 \\
& k^2(1-p) - 16p^{5/2}c = 0 \\
& e = \frac{k}{4p^{1/2}}; \ \beta = \frac{1}{p^{1/2}}
\end{aligned}
$$

ii) If $\frac{k^2\overline{\beta}^3\left(\overline{\beta}^2 - 1\right)}{16} \leq c \leq \frac{k^2\overline{\beta}^5}{2}$, *then*

$$
\begin{aligned}
& s = 0; \ \beta = \overline{\beta} \\
& \overline{\beta}^5 k^2 (1-p) - 2c\left(1 + p\overline{\beta}^2\right)^3 = 0 \\
& e = \frac{k\overline{\beta}}{2\left(1 + p\overline{\beta}^2\right)}
\end{aligned}
$$

iii) If $c \geq \frac{k^2\overline{\beta}^5}{2}$, *then*

$$
s = 0; \ p = 0; \ \beta = \overline{\beta}; \ e = \frac{k\overline{\beta}}{2}
$$

■

Note that in this case the firm decides to optimally underinvest in technology, that is, $\beta \geq 1$. We only have $\beta = 1$ when monitoring is costless, where the optimal probability of inspection is $p = 1$. Here, we also have that the pollution level is positively linked to the monitoring costs, that is, the more expensive the monitoring, the larger the pollution level, and viceversa. But, contrary to the three-stage game presented in the previous Section, we have that the technology investment is positively related to the monitoring costs also. This is so because in this case the firm cannot induce a lenient regulation with the selection of a more expensive technology and, therefore, the extra cost in investment cannot be compensated with smaller expected fines.

Observe that cases ii) and iii) of the Proposition are exactly the same as those of Lemma 2 when $\beta = \overline{\beta}$ and $\phi(\beta) = \beta$. Here, only when monitoring is expensive enough (case iii), the authority optimally decides to leave the firm unregulated, which results in $\beta = \overline{\beta}$ and $e = \frac{k\overline{\beta}}{2}$. These results are different from those of the three-stage game, in which the firm decides, in advance, a sufficiently expensive technology to induce a zero expected fine for noncompliance (see case ii of Proposition 3).

Now, we consider the solution of the problem when $\phi(\beta) = t$.

4.3 First stage when $\phi(\beta) = t$

Given the policy (s, p), the firm solves the following problem:

$$\begin{aligned} Max_{e,\beta} \quad & \left\{ ke - \frac{e^2}{\beta} - pt\,(e-s)^2 \right\} \\ s.a. \quad & s - e \leq 0 \\ & e - \frac{k\beta}{2} \leq 0 \\ & \beta - \overline{\beta} \leq 0 \end{aligned} \tag{18}$$

Here, the solution is given in the following:

Lemma 8 *The solution is:*

$$e = \frac{\overline{\beta}\,(k + 2pts)}{2\left(1 + pt\overline{\beta}\right)};\;\; \beta = \overline{\beta} \tag{19}$$

$$\lambda_1 = \lambda_2 = 0;\;\; \lambda_3 = \left(\frac{k + 2pts}{2\left(1 + \overline{\beta}pt\right)}\right)^2 > 0$$

■

Here, since the firm plays after the regulator and fines do not depend on the selected technology, the firm chooses the cheapest technology. Therefore, the solution to problem (18) is equivalent to that of Lemma 1, where $\beta = \overline{\beta}$ and $\phi(\beta) = t$.

4.4 Second stage when $\phi(\beta) = t$

The regulator's problem, considering (19), is the following:

$$
\begin{array}{ll}
 & Max_{s,p} \left\{ ke - \dfrac{e^2}{\overline{\beta}} - \overline{\beta} e^2 - pc \right\} \\
s.a. & 2p^{1/2}s - k \leq 0 \\
- & s \leq 0 \\
- & p \leq 0 \\
 & p - 1 \leq 0
\end{array}
$$

and the solution is identical to that of Lemma 2, where $\beta = \overline{\beta}$ and $\phi(\beta) = t$.

Therefore, in this case the regulator cannot presuade the firm to invest in a cleaner technology, since it does not provide the necessary incentives. Following a similar reasoning to that of the previous Section, it can be easily shown that the firm prefers that fines depend on the technology level, as in the three-stage game. However, the regulator in this case prefers that fines depend on the technology level, contrary to the three-stage game. The explanation fro this result comes from the fact that the regulator cannot control the technology investment if fines depend on noncompliance only, which may result in large environmental damages.

In the next Section, we analyze the results of both the three- and the two-stage games, for both penalty structures $\phi(\beta) = \beta$ and $\phi(\beta) = t$.

5 Discussion

In this Section, we compare the results of the solutions obtained in Sections 3 and 4 in terms of the firm's expected payoff and social welfare ((4) and (5), respectively). In Table 1 in the Appendix, we present their general expressions for the corresponding ranges of the monitoring costs, considering the two dimensions of our problem, that is, the fact that the firm may anticipate or not the investment in technology (i.e., a three-stage game versus a two-stage game) and that fines can depend or not on the amount of that investment (in our analysis, either $\phi(\beta) = \beta$ or $\phi(\beta) = t$). For instance, in the three-stage game with $\phi(\beta) = \beta$ (problem A, from now on) we consider the results presented in Corollary 4, suitably substituted in (4) and (5). In the three-stage game with $\phi(\beta) = t$ (problem B), we proceed in the same way considering the results of Corollary 5. In the two-stage game with $\phi(\beta) = \beta$ (problem C), we consider the results of Proposition 7 and, finally, in the two-stage game with $\phi(\beta) = t$ (problem D), we take the solution presented in Lemma 2, considering $\beta = \overline{\beta}$ and $\phi(\beta) = t$.

Considering the expressions in problem A, β_a refers to the optimal investment in technology obtained in Corollary 4. As for problem B, p_b and β_b are, respectively, the optimal probability of inspection and the level of technology obtained in Corollary 5. In problem C, p_c and β_c refer to the optimal probability and the technology level obtained in Proposition 7. Finally, in problem D, p_d is the optimal probability obtained Lemma 2, considering $\beta = \overline{\beta}$ and $\phi(\beta) = t$.

Due to the difficulty of comparing the results presented in Table 1, we have concentrated in the following example. Observe that the firm's degree of profitability (k) affects positively to both the firm's expected payoff and the social welfare in the four problems (k^2 is multiplying in the numerator in all the expressions). Therefore, from now on, and without loss of generality, we consider $k = 1$. As for the other parameters of the problem (t and $\overline{\beta}$), we take the case in which $t = \overline{\beta} = 2$. We consider both parameters equal to make the four problems equivalent from a sufficiently large level of the monitoring cost, as we will see later on.[14]

Considering the mentioned levels of the parameters, we have computed the corresponding values of the firm's expected payoff and the social welfare for alternative levels of the monitoring costs and the results are depicted in Figures 3 and 4, respectively.

Regarding Figure 3, we first observe that in the four problems the firm's expected payoff is positively related to the monitoring costs. This is so because, when the monitoring costs increase, the announced probability of inspection is smaller and, therefore, this is benefitial for the firm's own interest. We also see that the three-stage game is superior to the two stage game

[14]See Section 3, above, for a discussion on the repercussion of t and $\overline{\beta}$ being different on the firm's expected profits and social welfare.

from the firm's view point, independently of the selected structure of the fines (i.e., $\pi_a \geq \pi_c$ and $\pi_b \geq \pi_d$, for every c). This can be explained arguing that, in the three-stage game, the firm can affect the regulatory policy to its favor, anticipating the investment in technology. In Section 3, we obtained that this anticipation results in an over-investment in technology with respect to the efficient investment level, but this is more than compensated with a lenient regulation that allows for a saving in expected penalties. For example, in the three stage game, the optimal probability of inspection is zero if $c \geq \frac{\sqrt{2}}{16}$ when $\phi(\beta) = \beta$, and if $c \geq 1$ when $\phi(\beta) = t$. However, at those levels of the monitoring costs, the respective optimal probalities of inspection in problems C and D are $p_c = 0.60213$ and $p_d = 0.6984$ if $c = \frac{\sqrt{2}}{16}$, and $p_c = 0.28797$ and $p_d = 0.30737$ if $c = 1$. Also, as we discussed in the two previous Sections, the firm weakly prefers that fines depend on the investment in technology in both games, since the firm has more flexibility in this case to affect penalties to its favor. In the two-stage game, the firm strictly prefers $\phi(\beta) = \beta$ when $0 \leq c < 1$ (note that, when $c \geq 1$, both problems are equivalent since we are considering $\overline{\beta} = t$). In the three-stage game, the firm strictly prefers $\phi(\beta) = \beta$ when $0 < c < 16$. Finally, the results of the four problems are equivalent when $c \geq 16$, where the firm is unregulated and decides $e^* = 1$, $\overline{\beta}^* = 2$, resulting in $\pi^* = 0.5$ and $R^* = -1.5$.

Considering Figure 4, we observe that, obviously, social welfare is negatively related to the monitoring costs. Surprisingly, the three-stage game is also superior to the two-stage game from the regulator's view point (here, we have $R_a \geq R_c$ and $R_b \geq R_d$, for every c). The explanation for the result is that, in the former case, the regulator creates the firm's necessary inducement to invest in environmental technology with the promise of a lenient regulation, with the corresponding savings in monitoring costs and environmental damages. For instance, in problem A, the optimal probability of inspection is zero when $c = \frac{\sqrt{2}}{16}$, that results in $\beta_a = \frac{1}{\sqrt{2}}$, $e_a = \frac{\sqrt{2}}{4}$ and $R_a = 0.088$. However, at that same level of the monitoring cost, in problem C we have $p_C = 0.60213$, $\beta_C = 1.2887$, $e_C = 0.322175$ and $R_C = 0.04564$. Also, at $c = 1$, we have $p_b = 0$, $\beta_b = 1$, $e_b = \frac{1}{2}$ and $R_b = 0$, while $p_d = 0.30737$, $\beta_d = 2$, $e_d = 0,4485$ and $R_d = -0.3618$. Regarding the structure of the fines preferred by the regulator, we see that this depends on the game we consider. In the three-stage game, we observe that, in general, the regulator prefers $\phi(\beta) = t$ to $\phi(\beta) = \beta$. In both cases, the firm has incentives to invest in clean technology, but in the second case, the firm has more flexibility to avoid the penalty, with the appropriate overinvestment in the technology. However, in the two-stage game, the regulator prefers $\phi(\beta) = \beta$ to $\phi(\beta) = t$, since with the latter structure, the firm has no incentive to invest in clean technology, resulting in large environmental damages.

Summarizing, we conclude that both the firm and the regulator would agree that the three stage game is beneficial for both, that is, the fact that the firm may anticipate the investment in technology to the policy announcement. Therefore, this would result in inspections being very occasional and, therefore, expected penalties being very small. In fact, expected penalties are zero in problem A when $c \geq \frac{\sqrt{2}}{16}$, and in problem B when $c \geq 1$, while expected penalties are zero in problems C and D when $c \geq 16$. While it seems logical that the firm would prefer to anticipate to the policy announcement, however it is quite surprising that the regulator would also prefer that anticipation, since the subsequent penalties for noncompliance would be very small. The explanation for this result relies on the fact that the regulator induces the firm to overinvest in technology if it waits to announce the policy, obtaining small environmental damages with small expected monitoring costs. Finally, while the firm always prefers that fines depend on the investment in technology, the regulator would only prefer that possibility in the two-stage game, that is, when the firm postpone its investment decision to know the parameters of the environmental policy.

6 Conclusions

In this paper, we have presented a principal-agent model in which the regulator chooses the terms of the policy and the firm selects the pollution level and the investment in environmental technology. By considering fines contingent on that investment together with the degree of noncompliance, we have shown that the firm overinvests in clean technology when that investment is prior to the policy announcement; however, it underinvests in technology when the investment decision is taken after knowing the terms of the policy.

We have shown that the firm prefers that fines are contingent on the technology investment, independently of the timimg of that decision with respect to the policy announcement. However, that timing is relevant for deciding the best structure of the fines from the regulator's view point, since it prefers fixed fines when the firm decides the investment first, and contingent fines when the investment decision is taken afterwards.

Finally, we have found that both the firm and the regulator prefer that the firm anticipates its investment decision to the policy announcement. On the one hand, the firm obtains a lenient regulation overinvesting in pollution technology; on the other hand, society saves monitoring costs and environmental damages with that overinvestment.

Some extensions of this model may include the possibility of having violations to positive standards, an aspect normally observed in practise that cannont be explained here. Our results suggest that probabilistic taxes (i.e., standards equal to zero, with a positive chance of being discovered) are always better than positive standards from an optimal point of view, but that need not be necessaryly the case. We believe that some kind of asymmetry with respect to some characteristic of the firms, such as its profitability, may induce to having violations to positive standards. This is left for future research.

7 Appendix

The necessary optimality conditions for problem (14) are the following:

$$
\begin{aligned}
&\frac{\partial L}{\partial \beta} = \frac{k}{2}\left(\frac{\partial e}{\partial \beta}\right) - \gamma_1 - \gamma_2\left(\frac{\partial g}{\partial \beta}\right) - \gamma_3\left(p\phi' - 1\right) = 0 \\
&\frac{\partial L}{\partial p} = \frac{k}{2}\left(\frac{\partial e}{\partial p}\right) - \gamma_2\left(\frac{\partial g}{\partial p}\right) - \gamma_3\phi + \gamma_4 - \gamma_5 = 0 \\
&\gamma_1\left(\beta - \overline{\beta}\right) = 0 \\
&\gamma_3\left(-\beta + p\phi\right) = 0 \\
&\gamma_4 p = 0 \\
&\gamma_5\left(p - 1\right) = 0
\end{aligned}
$$

together with the five restrictions of problem (14), the nonnegativity of the Lagrange multipliers $\gamma_1, \gamma_3, \gamma_4$ and γ_5, and $\gamma_2 \in \mathbb{R}$.

Also, from (10) and (12), we have the following:

$$
\begin{aligned}
\frac{\partial e}{\partial \beta} &= \frac{k\left(1 - p\beta^2\phi'\right)}{2\left(1 + p\phi\beta\right)^2} \\
\frac{\partial e}{\partial p} &= -\frac{k\beta^2\phi}{2\left(1 + p\phi\beta\right)^2} \\
\frac{\partial g}{\partial \beta} &= k^2\beta^2\left[\left(3\phi + \beta\phi'\right)\left(\beta - p\phi\right) + \beta\phi\left(1 - p\phi'\right)\right] - 6cp\left(\phi + \beta\phi'\right)\left(1 + p\beta\phi\right)^2 \\
\frac{\partial g}{\partial p} &= -\beta\phi\left[k^2\beta^2\phi + 6c\left(1 + p\beta\phi\right)^2\right]
\end{aligned}
$$

TABLE 1. The firm's expected payoff and the social welfare in problems A, B, C and D.

	The three-stage game	**The two-stage game**
$\boldsymbol{\phi} = \boldsymbol{\beta}$	*i) If* $0 \le c \le \frac{k^2\sqrt{2}}{16}$, *then* $\pi_a = \frac{k^2}{8\beta_a}$; $R_a = \frac{k^2\left[\beta_a^4+\left(2\beta_a^2-1\right)^2\right]}{16\beta_a^3}$ *ii) If* $\frac{k^2\sqrt{2}}{16} \le c \le \frac{k^2\overline{\beta}^5}{2}$, *then* $\pi_a = \frac{k^2\beta_a}{4}$; $R_a = \frac{k^2\beta_a\left(1-\beta_a^2\right)}{4}$ *iii) If* $c \ge \frac{k^2\overline{\beta}^5}{2}$, *then* $\pi_a = \frac{k^2\overline{\beta}}{4}$; $R_a = \frac{k^2\overline{\beta}\left(1-\overline{\beta}^2\right)}{4}$	*i) If* $0 \le c \le \frac{k^2\overline{\beta}^3\left(\overline{\beta}^2-1\right)}{16}$, *then* $\pi_c = \frac{k^2\beta_c}{8}$; $R_c = \frac{k^2\beta_c\left(2-\beta_c^2\right)}{8}$ *ii) If* $\frac{k^2\overline{\beta}^3\left(\overline{\beta}^2-1\right)}{16} \le c \le \frac{k^2\overline{\beta}^5}{2}$, *then* $\pi_c = \frac{k^2\overline{\beta}}{4\left(1+p_c\overline{\beta}^2\right)}$; $R_c = \frac{k^2\overline{\beta}\left[\left(1-\overline{\beta}^2\right)\left(1+3p_c\overline{\beta}^2\right)+4p_c^2\overline{\beta}^4\right]}{4\left(1+p_c\overline{\beta}^2\right)}$ *iii) If* $c \ge \frac{k^2\overline{\beta}^5}{2}$, *then* $\pi_c = \frac{k^2\overline{\beta}}{4}$; $R_c = \frac{k^2\overline{\beta}\left(1-\overline{\beta}^2\right)}{4}$
$\boldsymbol{\phi} = \mathbf{t}$	*i) If* $0 \le c \le \frac{k^2 t}{2}$, *then* $\pi_b = \frac{k^2}{4(1+p_b t)}$; $R_b = \frac{p_b^2 k^2 t^2}{(1+p_b t)^3}$ *ii) If* $\frac{k^2 t}{2} \le c \le \frac{k^2\overline{\beta}^4 t}{2}$, *then* $\pi_b = \frac{k^2\beta_b}{4}$; $R_b = \frac{k^2\beta_b\left(1-\beta_b^2\right)}{4}$ *iii) If* $c \ge \frac{k^2\overline{\beta}^4 t}{2}$, *then* $\pi_b = \frac{k^2\overline{\beta}}{4}$; $R_b = \frac{k^2\overline{\beta}\left(1-\overline{\beta}^2\right)}{4}$	*i) If* $0 \le c \le \frac{k^2\overline{\beta}^4 t}{2}$, *then* $\pi_d = \frac{k^2\overline{\beta}}{4\left(1+p_d\overline{\beta}t\right)}$; $R_d = \frac{k^2\overline{\beta}\left[\left(1-\overline{\beta}^2\right)\left(1+3p_d\overline{\beta}t\right)+4p_d^2\overline{\beta}^2 t^2\right]}{4\left(1+p_d\overline{\beta}t\right)}$ *ii) If* $c \ge \frac{k^2\overline{\beta}^4 t}{2}$, *then* $\pi_D = \frac{k^2\overline{\beta}}{4}$; $R_D = \frac{k^2\overline{\beta}\left(1-\overline{\beta}^2\right)}{4}$

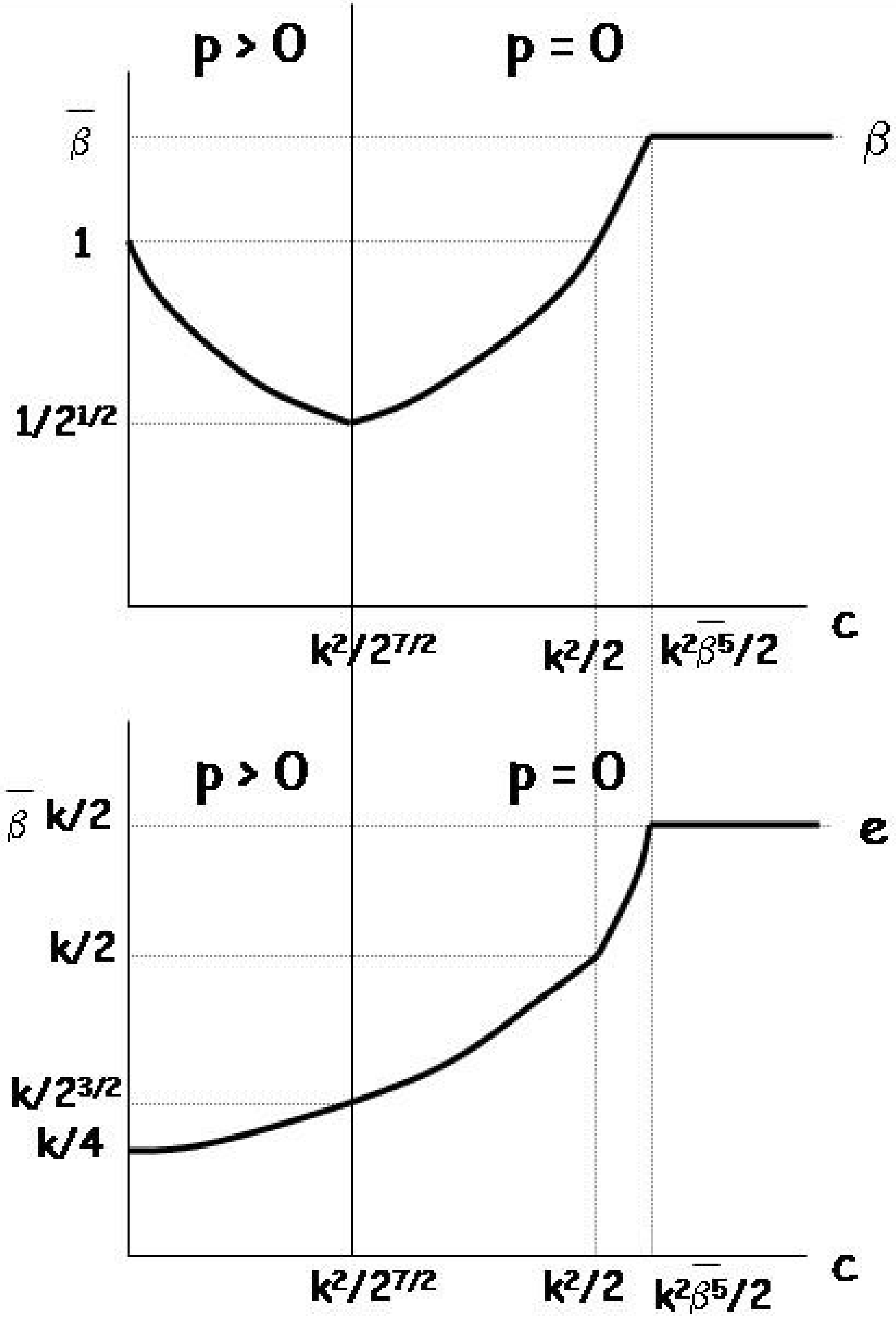


Figure 1. The optimal choice of the firm in the three – stage game when $\phi(\beta) = \beta$

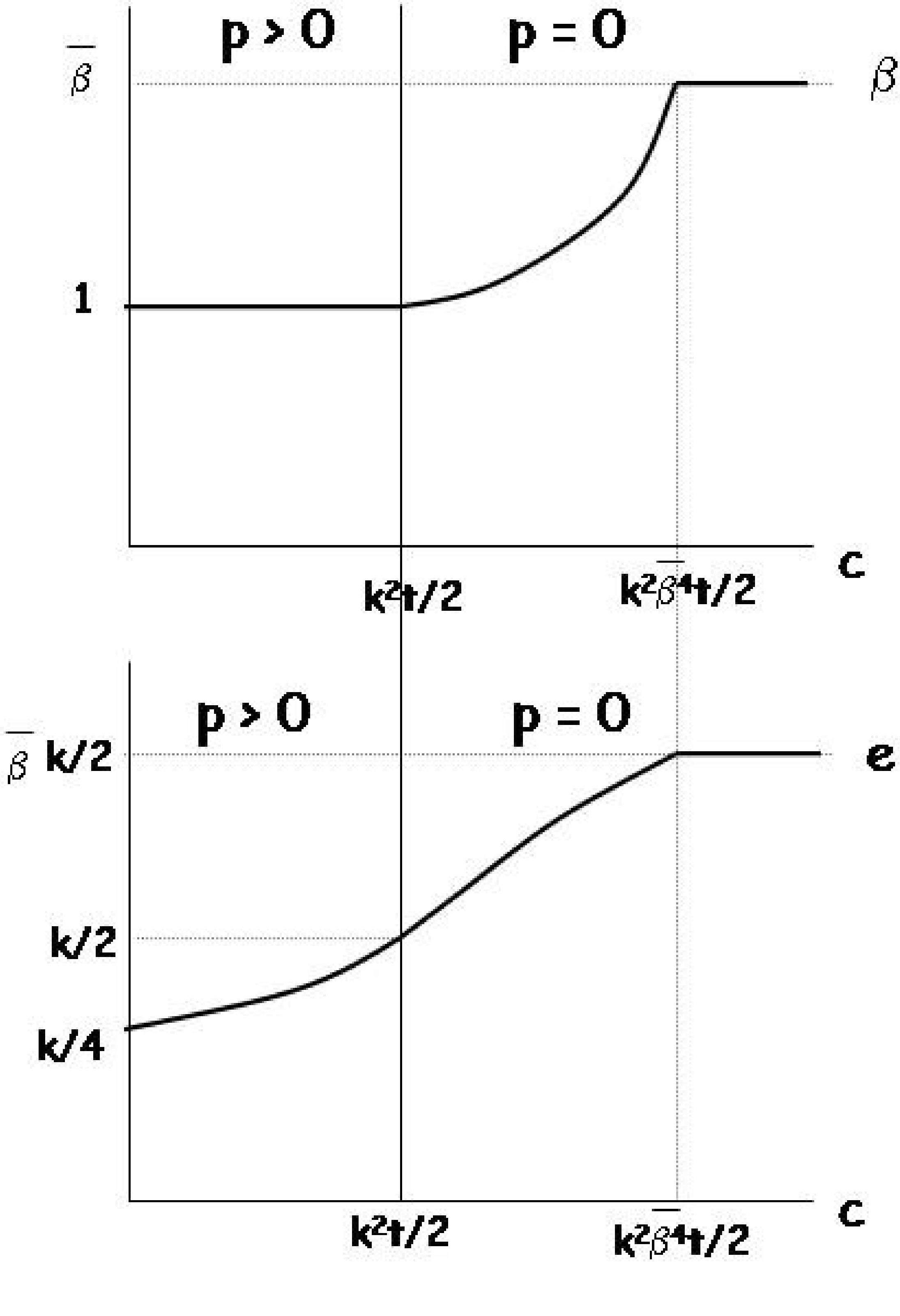


Figure 2. The optimal choice of the firm in the three – stage game when $\phi(\beta) = t$

Figure 3. The firm's expected payoff as a function of the monitoring cost

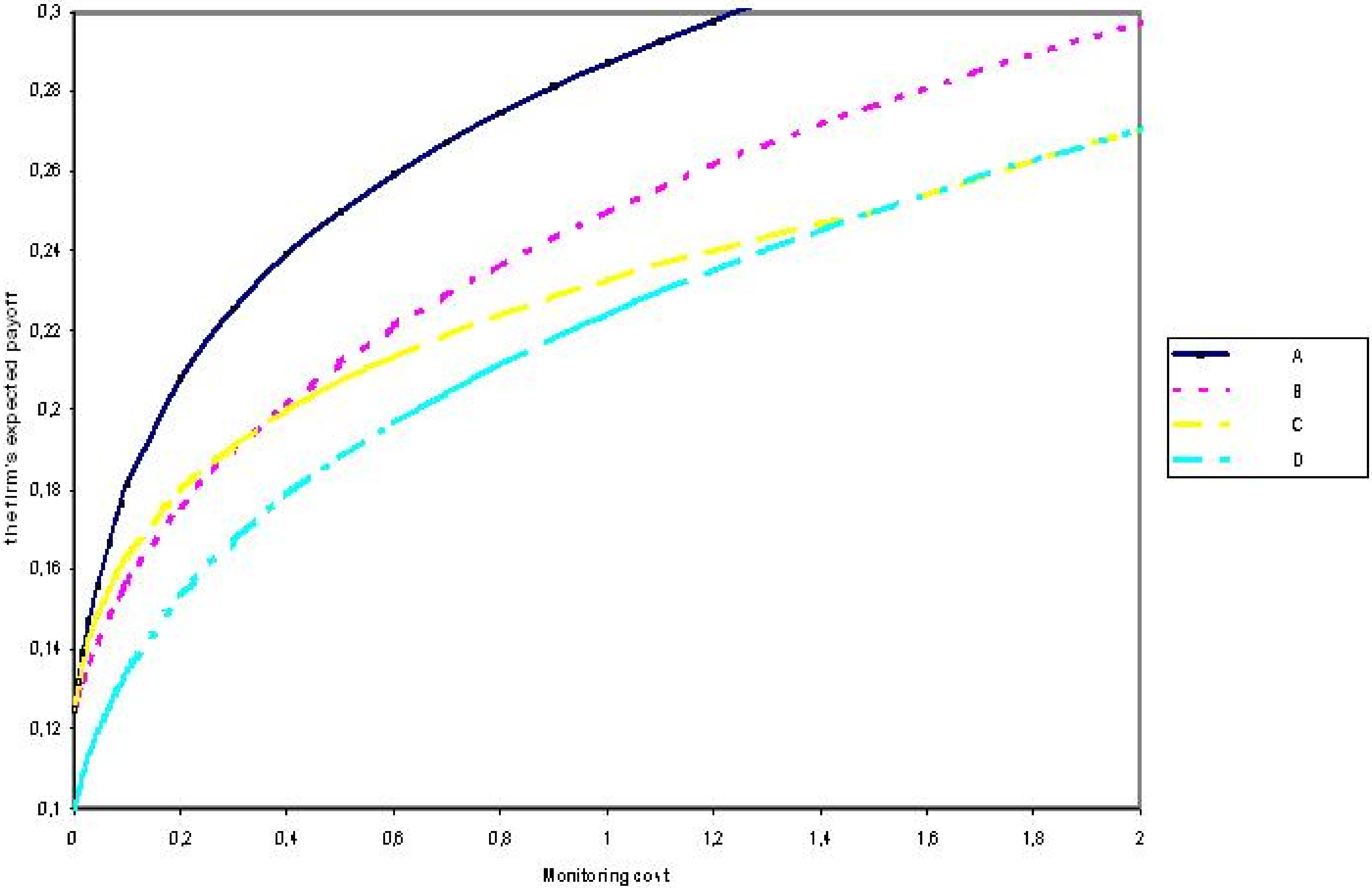
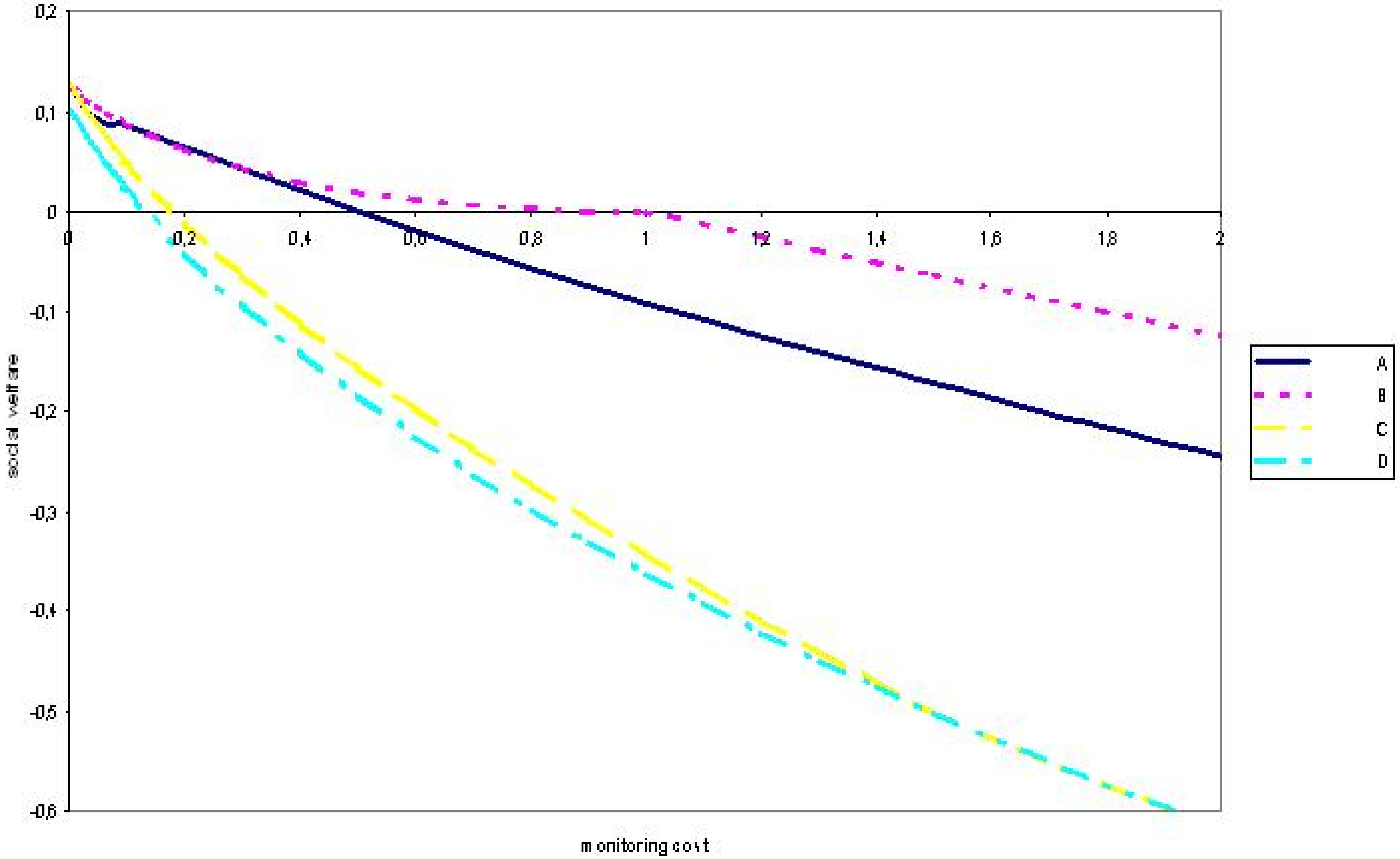


Figure 4. The social welfare as a function of the monitoring cost



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